

**THE EFFECT OF USING VAN HIELE'S INSTRUCTIONAL MODEL IN THE
TEACHING OF CONGRUENT TRIANGLES IN GRADE 10 IN GAUTENG HIGH
SCHOOLS.**

Submitted by: **Sadiki Muraga William Riizo**

3663 6479

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No achievement in life is without the help of many known and unknown individuals who have contributed to our lives. We are all the sum total of what we have learnt from others, and we owe any measure of success to the array of input from so many. Here are just a few, who made this work possible:

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DECLARATION OF ORIGINALITY

Name: SADIKI MURAGA WILLIAM RIIZO

Student number: 3663 6479

Degree: MASTER OF SCIENCE IN MATHEMATICS EDUCATION

THE EFFECT OF USING VAN HIELE'S INSTRUCTIONAL MODEL IN THE TEACHING OF
CONGRUENT TRIANGLES IN GRADE 10 IN GAUTENG HIGH SCHOOLS

I declare that the above dissertation/thesis is my own work and that all the sources
that I have used or quoted have been indicated and acknowledged by means of
complete references.

.....

(Signature)

(MURAGA WILLIAM RIIZO SADIKI)

.....

(Date)

ABSTRACT

This study investigates the effect of using Van Hiele's instructional model in the teaching of congruent triangles in grade 10 in Gauteng high schools. Three randomly selected high schools in Gauteng formed the research fields, while intact groups of grade 10 learners in these schools formed the study participants (136 learners) for the study.

A mixed method approach which was adopted for the study, using pre-test/post-test matching control group design and classroom observation. The pre-test/post-test was used to collect quantitative data, while classroom observation was used to glean qualitative data. Some of the findings from the quantitative data analysis suggested that the intervention improve the achievement scores in the experimental groups while the qualitative data was revealed that the intervention facilitated the learning of the concepts of congruent. It was recommended that Van Hiele learning and instructional model be adopted and applied in the teaching of other areas of mathematics.

ACRONYMS

ANAs	Annual National Assessments
CAPS	Curriculum Assessment Policy statements
DoE	Department of Education
ES	Effect size
FET	Further Education and Training
GET	General Education and Training
NCS	National Curriculum Statements
RFP	Research Field Profile
VHIA	Van Hiele Instructional Approach

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CHAPTER ONE

1. Background of the Study

1.1 Introduction

This study IS inspired by the high failure rate of learners in mathematics in the matriculation examination in South Africa (DoE, 2012). The set of 2014 matriculates' were the first learners that matriculated under the new "Curriculum Assessment Policy Statement" (CAPS) curriculum. The Department of Education (DoE) has reported that, compared to those who matriculated under the previous curriculum, the pass rate has not improved (DoE, 2015).

The poor performance of matriculating learners in mathematics over the years has been a serious concern to all education stakeholders, particularly the government, who surely want to show the country that they have put a working educational policy in place. All the efforts of the government and academic society to improve student performance in matriculation mathematics results have yielded little noticeable results. In fact, according to the Trends in International Mathematics and Science Study (TIMSS) report (Gonzales, 2009) South African learners ranked lowest in the world.

The 2010 annual national assessments (ANAs) argue that poor performance stems from primary school. Furthermore, the 2012 ANAs report revealed that two thirds of pupils in Gauteng leave primary school without the adequate conceptual knowledge necessary to succeed in in secondary school mathematics (Adler and Sfard, 2015). These learners battle to understand mathematical concepts throughout the high school grades.

Matric mathematics exams include algebra, financial mathematics, trigonometry, calculus and geometry. Geometry makes up about 60% of the paper 2 of the matric mathematics examination . Although it is not possible as at the time of this study to numerically assess the extent to which geometry contributes to the failure rate of mathematics matriculates', both Van Hiele (1986) and French (2004) link students'

ability in mathematics to their level of geometrical conceptual ability. However, study (Atebe, 2008) has shown that both teachers and learners mystify the geometry aspect of the mathematics curriculum, and that South African learners are experiencing conceptual learning difficulty in geometry, (De Villiers, 1997; Brombacher, 2001; Howie, 2001; Roux, 2003).

It is therefore possible that the continual poor performance of South African learners in mathematics is caused by their poor knowledge of geometry. Van der Sandt (2007) describes the learning of geometry in South African high schools as problematic. Geometry concepts entail deductive reasoning of proofs and representative diagrams. Solving geometrical problems is largely based on the learners' competency to apply the definitions, axioms, postulates, theorems and proofs to solve problems. Van Hiele (1999) and Atebe (2008) argue that students are often lazy to think deeply and therefore find geometry learning difficult.

Over a decade ago, De Villiers (1997) remarked that "unless we (South Africans) embark on a major revision of the primary school geometry curriculum along Van Hiele lines, it seems clear that no amount of effort at the high school will be successful". De Villiers revisited and confirmed this in 2006 (De Villiers, 2004). There are other scholars (Brombacher, 2001; Howie, 2001, Roux, 2003) who share the same opinion.

Despite de Villiers and other scholars' concern over the teaching and learning of geometry in South African schools, the school curriculum is still not aligned to the Van Hiele geometry learning theory. The post-apartheid mathematics curriculum in South Africa will be briefly reviewed in the next section of this thesis.

Based on the above, the effect of the Van Hiele instructional model on the teaching of congruent triangles will be investigated in this study.

1.1.1 Curriculum Policies in South Africa from 1994 to the Present Time

When the new, democratically elected government took over in South Africa in 1994, the Department of Education (DoE), was formed (Chisholm, 2003). In 1997, the DoE implemented a curriculum tagged C2005, which was based on the outcomes-based

education policy (DoE, 2002). In the year 2000, C2005 was reviewed and the revised edition of the C2005 was named National Curriculum Statement (NCS), (DoE, 2002). In 2008, the DoE revisited the issue of curriculum review and the revised edition of the review of NCS was named Curriculum Assessment Policy Statement (CAPS) (DoE, 2012). CAPS was to be implemented in three phases: the first phase in 2012 with Grades 1, 2, 3 & 10; followed by Grades 4 to 6 and 11 in 2013 and lastly, Grades 7 - 9 and 12 in 2014 (DoE).

1.1.2 The Mathematics aspect of the National Curriculum Statement (NCS)

The South African CAPs for high school mathematics stresses four major learning components to be taught and learned in senior secondary mathematics. These learning components are referred to as 'Learning Outcomes' (South Africa, DoE, 2003, p.7). The learning outcomes as set out in the CAPs are as follows:

- Learning Outcome 1: Number and Number Relationships.
- Learning Outcome 2: Functions and Algebra.
- Learning Outcome 3: Space, Shape and Measurement.
- Learning Outcome 4: Data Handling and Probability.

This study will focus on learning outcome 3, which is concerned with the geometry in the mathematics curriculum. It comprises of analytical and Euclidean geometry. The geometry aspect of the matriculation examination is done in paper 2, and forms 60% of the total mark in this paper: analytical geometry contributes 27%, while Euclidean geometry takes 33%. The DoE (2003) stresses the development of students' skills in making and testing conjectures, investigating, justifying, proving, and generalizing Euclidean geometry. Given the emphasis on these skills in the CAPs, the objectives of high school (Euclidean) geometry teaching in South Africa may be summarized as follows:

- ♦ Development of students' spatial awareness and visualization abilities through the use of various methods, including geometrical constructions, to investigate geometrical properties of two- and three-dimensional figures.

- ♦ Development of students' reasoning abilities through explicit teaching of processes such as experimentation, testing conjectures, congruency and justifying statements that would ultimately lead to the acquisition of skills in proof writing in Euclidean geometry.
- ♦ Development of students' problem solving abilities by using geometrical properties to solve a wide range of problems in many other aspects of mathematics, such as trigonometry, algebra, and other related fields.

This study investigates the learning of geometrical concepts through the Van Hiele learning model.

1.1.3 The Mathematics aspect of the Curriculum Assessment Policy Statement (CAPS)

In the CAPS curriculum, every subject in each grade has a single, comprehensive and concise policy document, which specifies what has to be taught, when it has to be taught, and how it should be assessed (DoE, 2012.). The terminology "learning outcomes" and "assessment standards" in the NCS were replaced with "content and skills". In the Foundation Phase (Grades R, 1, 2 & 3), "numeracy" is now called "mathematics".

Comparison between Mathematics Assessment Areas in the NCS and CAPS

Table 1

Serial Number	NCS	CAPS
Paper 1	Algebra, Sequences, Functions, Calculus, Linear Programming, Finance	Algebra, Sequences, Functions, Calculus, Finance, Probability
Paper 2	Coordinate geometry, Transformation geometry, Trigonometry, Statistics	Coordinate geometry, Euclidean geometry, Trigonometry, Statistics (including regression and correlation)
Paper 3	Euclidean geometry, Probability and Statistics Note: This paper is done by less than 5% of the matric mathematics students	

Table 1 shows the main differences between the NCS matric assessment areas and that of the CAPS.

Table 2 : Comparing the subtopics between the NCS and CAPS

NUMBER OF SUBTOPICS		
	NCS	CAPS
FUNCTIONS	52	49
PATTERNS & SEQUENCES	10	10
FINANCE	10	9
ALGEBRA	27	26
CALCULUS	14	13
PROBABILITY		19
EUCLIDEANGEOMETRY	7	32
ANALYTICAL GEOMETRY	8	9
TRIGONOMETRY	16	15
STATISTICS & DATA HANDLING	15	19
TRANSFORMATION GEOMETRY	6	
LINEAR PROGRAMMING	8	
Total number of Subtopics	173	201

Table 2 compares the number of subtopics involved in both the NCS and CAPS (Umalusi, 2014).

It is clear from table 1 that all mathematics learners are compelled to do Euclidean geometry and table 2 shows that much more Euclidean geometry was introduced into the CAPS curriculum. This decision might have been motivated by the continual poor performance of mathematics matriculates' and the established link between performance in geometry and other areas of mathematics as discussed under subsection 1.1 above. Euclidean geometry has to do with taking intuitive axiomatic

assumptions for a small set and thereafter use these axioms to deduce further proposition or theorems.

In fact, geometry requires deeper reasoning than any other aspect of mathematics (Atebe, 2008). It is one of the most difficult aspects of mathematics to teach and difficult to learn for the students as well. This is the reason why this study will investigate the use of another teaching approach, apart from the conventional chalk-and-talk method of knowledge dissemination, to improve the teaching and learning of geometry. Due to the fact that grade 10 belongs to the first phase of CAPS implementation (introduced in 2012), and the pre-study information gathered by the researcher showed that learner's find the learning of the concepts of congruent triangles and its application difficult, it was decided to use grade 10 learners as study subjects. The study was underpinned by the Van Hiele learning and instructional approach.

1.2 Van Hiele's Geometry Learning and Instructional Model

The van Hiele leaning model takes into cognisance how learners progress in geometric cognitive thinking and hence prescribe a way to present classroom instruction to meet their thinking ability at each level of the learners. The prescribed method of instruction is named "van Hiele' Instructional model". These learning and instructional models are different from other learning or instruction models like the "Constructivism" learning theory. The constructivism learning theory is about motivating or helping the learners to construct their own cognitive knowledge but van Hiele looks at how learners develop cognitive knowledge (in geometry) at their own pace and the instructional approach that will accommodate such level of thinking.

1.2.1 Van Hiele's Learning Model

The theory was proposed by Pierre van Hiele and his wife as a result of their PhD programme. The model consists of five levels which the van Hieles postulated to describe how children learn to reason in geometry. The theory stresses that students cannot be expected to prove geometric theorems until they have built up an

extensive understanding of the systems of relationships between geometric ideas.

The five levels are:

Level 1 Visualization

Learners can only recognize geometric figures on the basis of their appearance, and not their properties.

Level 2 Descriptive/Analytical

Learners are able to discuss the properties of basic figures and recognize them by these properties.

Level 3 Abstract/Relational

At this stage, the properties of shapes are ordered. Learners can understand and form an abstract definition, distinguish between necessary and sufficient conditions for a concept, and understand the relationship between shapes.

Level 4 Deduction

Learners understand the meaning of deductions. The object of thought is deductive reasoning, they can form a system of formal proofs, understand the roles of undefined terms, definitions, axioms, and theorems in Euclidean geometry, but they do not yet know any non-Euclidean geometry.

Level 5 Rigor

Students at rigor stage understand geometry at the level of mathematicians. They understand that definitions are arbitrary and do not actually need to refer to any concrete realization.

Van Hiele (1959) remarked that the above levels cannot be learned by rote, nor acquired by age, but rather be developed through familiarity, by experiencing numerous examples and counter examples, the various properties of geometric figures, the relationships between the properties, and how these properties are ordered. The five levels proposed by the Van Hieles describe how students advance through this understanding.

The research targets the grade 10 learners in Gauteng high schools. Alex and mammen (2012) suggests that grade 10 learners should be conveniently placed on Van Hiele's geometric learning level 3 (informal deduction).

1.2.2 Van Hiele Instructional model

When Van Hiele suggested a geometrical learning model, she realized that, in order to master the intended learning levels as prescribed in the model, an appropriate pedagogical approach would be required. She therefore suggested a series of instructional approaches that could facilitate the learning of geometry. Van de Walle (2004) notes that Van Hiele clarified the specific activities expected of the teacher while teaching geometry.

Below is the instructional model as compiled by www.eric.ed.gov accessed on 2nd February, 2015. The instructional steps were made up of five steps which were to ensure that students move from one Van Hiele learning level to a higher one in their geometric thinking:

"In the Van Hiele model an instructional plan, which is composed of five steps, was formed in order to provide a transition from one level to another in students' geometric thinking." (Crowley, 1987; Erdoğan, Durmuş & Bekci, 2007).

(i) Interview (research): The first step is the step in which the geometric thinking levels of students are determined. In this step, the students' geometric thinking levels are determined through communication between the teacher and the student.

(ii) Direct Orientation: In this step, the teacher gives instructions and assignments related to the studies which will be done in the light of the answers he gets from the students. The purpose of the teacher giving assignments is to make students explore the structures about the topic by means of research.

(iii) Making clear (explanation): Teacher introduces the topic to students in this step and students combine their experiences with the words they used related to the topic. In this step, it is important for the teacher to arouse students' interests.

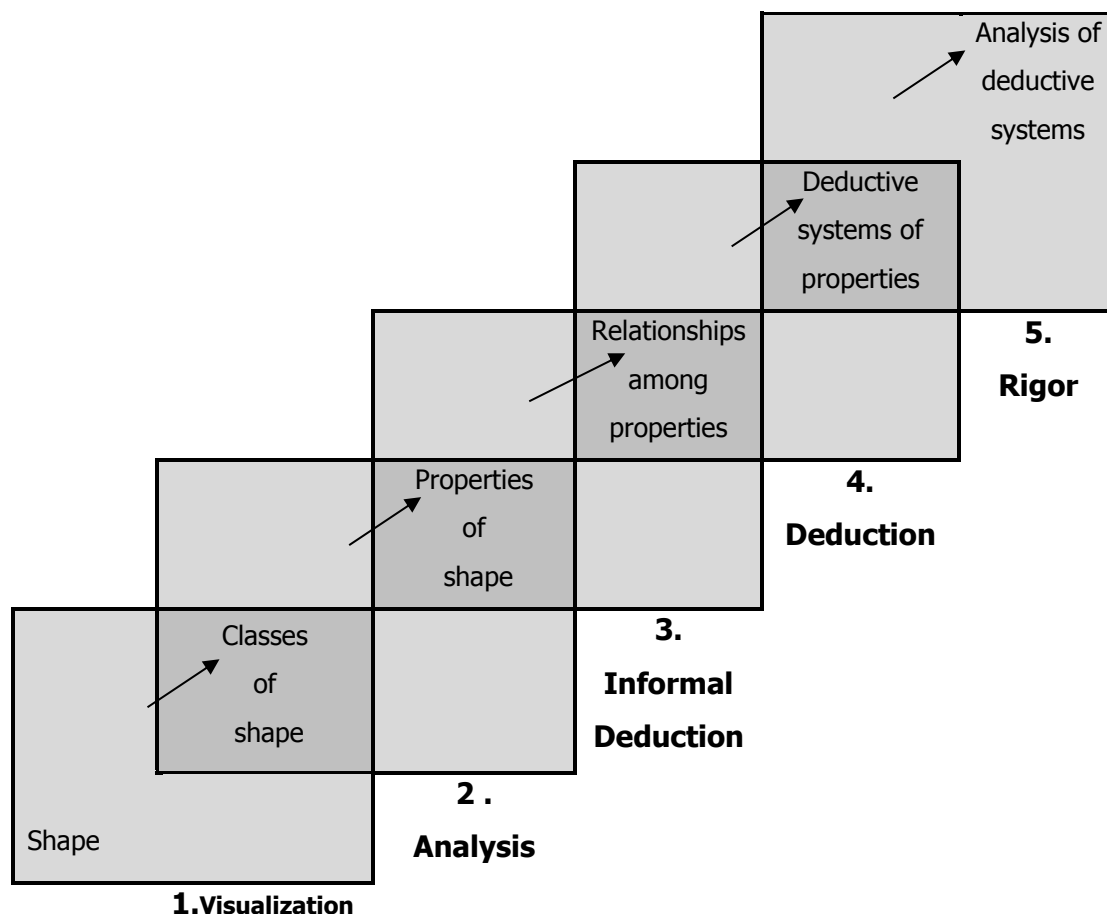
(iv) Free Performance (activities): Students work on different solutions of multiphase problems in this step. They discover the relationships ERDOĞAN, AKKAYA, ÇELEBİ AKKAYA/the effect of the Van Hiele Model based among the various objects of the structure in the topic they work on. The teacher should guide students in their thinking about different solutions.

(v) Integration: This step is the step in which students summarize and gather what they learned. Students internalize what they learned as a new thinking structure." www.eric.ed.gov.

This was visually structured by John A. van de Walle (2004), with a view to summarize the instructional models stated above. See figure 1 below.

Figure 1 Visual Display of Van Hiele Instructional Model

Below is the figurative expression of the instructional model



In this study, the Van Hiele instructional model described above will be adopted in the teaching of congruency. The instructional strategy was used to compile a checklist used to collect data during the classroom observation.

1.2.3 Learners and the Van Hiele Learning Model

Geometry requires critical and creative thinking. Generally, most students are lazy to think. Faleye and Mogari (2012), discovered that learners are unable do basic arithmetic calculations without the use of a calculator. Since geometry concepts are rooted in deep critical thinking, the question remains: how do learners who cannot calculate basic arithmetic successfully, engage in abstract concepts that need critical thinking? The findings of Faleye and Mogari (2012) might actually mean that South

African mathematics learners are lazy to think deep perhaps as a result of their poor mathematics background. In addition, Halat (2008) found out that the majority of high school learner's are on the first or second levels of the Van Hiele learning model (see subsection 1.1.3), whereas learners in grade 12 are supposed to be on level five. Again, this might be as a result of the poor thinking ability of the South African mathematics learners.

1.2.4 Adaptation of the Van Hiele Learning Model to the teaching of Congruent of Triangles in this study

In this section the van Hiele instructional model is adapted to the teaching of congruent of triangles. The steps highlighted below are as prescribed in the van Hiele instructional model.

- (i) The prior knowledge of the study participants are determined by asking probing questions on the concepts of the congruent of triangles to be taught. This is to determine the van Hiele geometric thinking level of the study participants.
- (ii) The study participants will be giving classwork on the foundational concepts of the congruent of triangles. This is to make the study participants think deeply, interact with each other and to consult available sources (research) about the concepts of the congruent of triangles.
- (iii) The topic of the day on the aspect of the congruent of triangles to be taught will be explained while drawing from the responses of the study participants given in the classwork in step (ii) above.
- (iv) More advance exercises that will enable relationship among objects and structures will be given in form of classwork or homework will be given.
- (v) The study participants are allowed to engage their answers to the exercises given in (iv) by explaining to the whole class. This step is to allow the study participants to be able to summarise and internalise the concepts learnt.

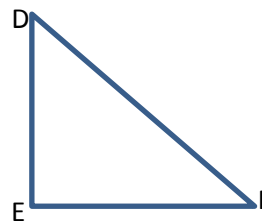
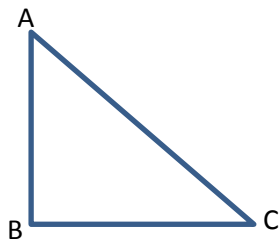
1.3 Congruent Triangle

1.3.1 The Concept of Congruent Triangles

When two triangles are equal in all respects, they are said to be congruent. Their angles are equal, their sides are equal and they can be placed exactly on top of each other. Their areas are also equal.

1.3.2 Conditions for Congruency

Given two triangles $\triangle ABC$ and $\triangle DEF$.

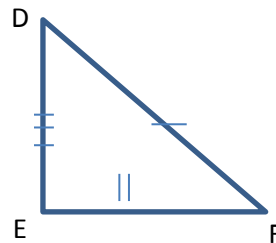
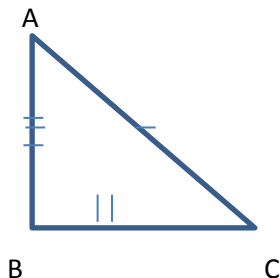


If $\triangle ABC$ is congruent to $\triangle DEF$, we write

$$\triangle ABC \equiv \triangle DEF$$

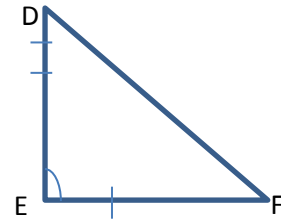
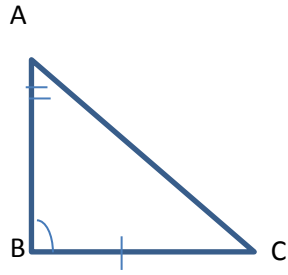
However, $\triangle ABC \equiv \triangle DEF$ only holds:

- (i) If the three sides of $\triangle ABC$ are equal to the three sides of $\triangle DEF$, this condition is written as SSS.

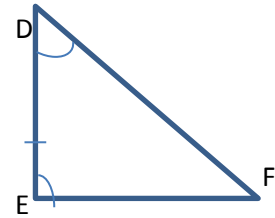
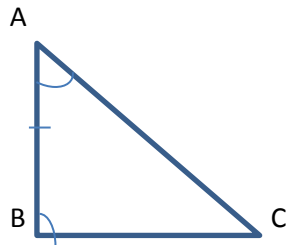


- (ii) If two sides of $\triangle ABC$ are equal to two sides of the $\triangle DEF$ and the angle included by those two pairs of equal sides are equal, this condition is written as SAS.

(iii)

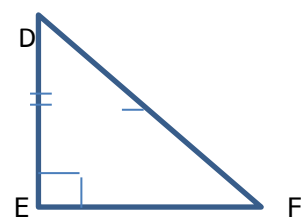
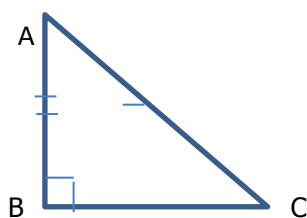


- (iv) If two angles of $\triangle ABC$ are equal to two angles of the $\triangle DEF$ and any one side of $\triangle ABC$ is equal to the corresponding side of $\triangle DEF$, this condition is written as SAA.



- (v) If two right angled triangles have equal hypotenuses and one other side of the triangle equal to one other side of the other triangle, it is written as RHS.

(vi)



The congruent concepts will be taught and tested on Van Hiele's geometric learning level 3.

This study is underpinned

1.4 Aim of the Study

The aim of this research work is to inquire the possible effect of teaching geometrical congruency to study participants who are in grade 10 in some of the Gauteng high schools, using van Hiele's instructional model (as the intervention).

The two dimensions of 'conceptual understanding' that will be measured are learning facilitation and performance. This does not mean to imply that the dimension of 'conceptual understanding' is limited to learning facilitation and performance alone.

1.5 Significance of the study

This study encourages the development of sound geometry understanding. Besides the fact that geometrical concepts carry a weight of 30% in the matric CAPS curriculum, sound knowledge of geometry may go a long way in solving the continual poor performance of matriculates' in mathematics. Studies have shown that sound knowledge in geometry facilitates conceptual understanding in other areas of mathematics (De Villiers, 1997).

In addition to this, sound geometrical skills have a wide application in other fields of life. Geometrical concepts are applicable in a wide variety of fields, such as information technology (IT) networking, navigation careers, oil fields, town planning etc. It is therefore crucial that learners' performance in geometry improves.

1.6 Problem of the Study

The problem of this study is to investigate the impact of teaching geometrical congruency using van Hiele's instructional model. The researcher hopes to achieve this by finding answers to the following questions:

1. How does the Van Hiele instructional model facilitate the learning of congruent triangle concepts in the participating high schools in the experimental schools?
2. How does the Van Hiele instructional model impact on study participants' score achievement in the learning of the concepts of congruent triangles in the experimental schools?

A null hypothesis, stated at 0.5 probability significant level shall be used to corroborate the answer/s to research question 1.

1.6.1 Hypothesis:

H₀: There is no statistically significant difference in the study participants' pre-test mean achievement score compared to the post intervention mean achievement score.

H₁: There is a statistically significant difference in the study participants' pre-test mean achievement score compared to the post intervention mean achievement score.

1.7 Definition of key terms.

1. ANAS: Annual National Assessments Standards
2. DoE: DEPARTMENT OF EDUCATION
3. Conceptual Understanding: In this study, conceptual understanding shall refer to improved scores in the summative test.
4. NCS: NATIONAL CURRICULUM STATEMENT
5. GET: GENERAL EDUCATION AND TRAINING
6. FET: FURTHER EDUCATION AND TRAINING
7. CAPS: CURRICULUM AND ASSESSMENT POLICY STATEMENTS
8. VHIA: VAN HIELE INSTRUCTIONAL APPROACH
9. ES: EFFECT SIZE
10. RFP: RESEARCH FIELDS PROFILE

1.8 The Layout of the study

Chapter one

Chapter presents the background, significance and the problem to the study.

Chapter Two

Chapter two reviews some of the previous, similar studies on the application of the Van Hiele learning theory in the teaching and learning of geometry in South African high schools.

Chapter Three

Chapter three describes the mixed method approach adopted in the study. It also describes the selection of the study participants, the research instruments and the development thereof, data collection and data analysis techniques, and ethical issues involved.

Chapter Four

The results obtained from the data analysis are presented in this chapter. These include both results from qualitative results (of the classroom observation data analysis) and quantitative results (of the hypothesis testing).

Chapter Five

Chapter 5 contains the discussion of major results emanating from the data analysis in view of the literature reviewed in chapter 2 of this study. The implications of the various results are also discussed.

Chapter Six

Chapter 6 presents the conclusion and recommendations emanating from the study.

CHAPTER TWO

2.0 Literature Review

2.1 Conceptual Framework

The high failure rate of matriculantes' in mathematics in South Africa repeats itself every year (Roux, 2003). According to Bombacher (2001) and Howie (2001) the results of the Third International Mathematics and Science Study-Repeat (TIMSS-R) conducted by the Human Sciences and Research Council (HSRC) in 1999 showed that out of the 38 countries that participated, South African learners obtained the poorest results in mathematics. It has clearly been a problem for many years.

Perhaps the problem stems from the "Segregation and Bantu" educational policy of the apartheid government of the past in South Africa. The democratically elected government of 1994 replaced the then apartheid government which put an end to the "Segregation and Bantu" policy. The policy was in force for a long time, hence it should be expected that it might take a while for its negative impact to be completely eradicated. Howie (2001, p.11) mentioned that South African children in the TIMSS-R had "considerable difficulty dealing with geometry questions and in some cases were successfully distracted by questions testing misconceptions" in geometry.

When learners are easily distracted in class, it implies that they have lost interest in whatever they are doing. It also implies that learning is not effective. For learners to gain conceptual understanding, they need to pay attention and be focused on what they are learning. This view was supported by behavioural psychologist Albert Bandura (1977), who conducted an analysis of behavioural learning, in which he found that learning involves four phases: attention, retention, reproduction, and motivation (Slavin, 1996).

Conceptual retention happens when a learner can reproduce what was learnt in the class at any time after the teaching period. In this situation, a learner demonstrates deep conceptual understanding of the topic that was taught and should be able to

apply the concept to solve problems. Maybe this is what the researchers (Bombacher, 2001; Howie, 2001 and Slavin, 1996) are calling the attention of education stakeholders to.

On the other hand Faleye and Mogari (2012) argue that the introduction of calculators from an early school age negatively impacts learners' mastery of the fundamental concepts of arithmetic computations and ability to solve arithmetic problems. The duo argues that the use of calculators from an early age may cause learners to become lazy to think deeply when solving mathematical problems.

Furthermore, good classroom and logical lesson presentation may arrest the attention of learners. Bandura, in his work, developed a learning model called Zone of proximal development (Bandura, 1977 in Slavin, 1996). This model emphasises a presentation of learning materials at or slightly above what learners are capable to do. Bandura explains that if learners are taught at a level either below or too far above their zone of proximal development, no learning will take place.

Geometry, a branch of mathematics, requires abstract thinking. This branch of mathematics is dreaded by many learners. Learners that are distracted or disturbed for no apparent reason usually indicates that the teacher failed to use an appropriate teaching approach, which means that learning is ineffective. A pedagogical approach that it is effective in the teaching of Algebra (another branch of mathematics) may also not be effective in the teaching of geometry (Noraini, 2005). In geometry in particular, Chop-koh (2000) and Halat (2008) advise that teachers need to plan classroom activities in a way that can help the learners understand the nature and the concepts of geometry. For example, De Villiers (1997) noted the importance of zone of proximal development in the Van Hiele's geometric learning model and said that "... Van Hiele reasoned that the failure of the traditional geometry curriculum resulted from teachers presenting the subject at a thought level higher than that of students."

Pierre and Dina van Hiele in their theses (1957 and 1986) and Halat (2008) underlined the idea that geometry concepts knowledge transfer should be systematic to such extent that the concepts are ordered. They argued that geometrical concepts should be logically presented to learners in such a way that the prior geometrical conceptual level of the learners is respected (Van Hiele, 1986). Hence, Van Hiele first identified the learning levels in geometry (see subsection 1.2.1). Based on this learning model, Van Hiele proposed instructional approaches that may help teachers to teach geometry according to the propagated conceptual level.

Studies (Chop-koh, 2000; Chew, 2009; Abdul Halim A'bdullah and Effandi Zakaria, 2011) support the notion that the Van Hiele classroom geometrical instructional approach impacts positively on students' geometric conceptual understanding and their level of reasoning. Perhaps this is why de Villiers (1997 and Roux (2008) suggested that the teaching of geometry in South Africa should follow van Hiele instructional model. Selected studies that have been carried out on the Van Hiele learning and instructional model are discussed under the subheadings.

Many investigations have been carried out on the applications of the van Hiele learning theory and its accompanied instructional model in the classroom teaching of geometry. Many aspects of geometry have been covered, but the researcher could not get a study carried out on its application on congruent triangles. Hence, this study looks at the application of van Hiele learning theory on the classroom teaching of congruent triangles.

2.2 Review of Relevant Literature Sources

2.2.1 The use of the Van Hiele models to improve the Geometrical conceptual thinking level in the classroom.

Chew Chew Meng (2009) investigate form one students' learning of solid geometry in a phase-based instructional environment using Geometer's Sketchpad (GSP) based on the Van Hiele theory. Chew examined students' initial Van Hiele levels of geometric thinking about cubes and cuboids, and their Van Hiele levels. He found

that study participants using the Van Hiele levels either increased or remained the same.

In another study, Erdogan et al., (2009) investigated the effect of the Van Hiele instructional model on the creative thinking level of 6th grade primary school learners by using the pre-test, post-test matching control group quasi-experimental design. The experimental group was taught geometry based on Van Hiele instructional model while the control group was taught geometry with the traditional method. Among other findings, they reported a statistically significant difference between the pre- and post-test creative thinking of study participants in the experimental groups.

A study that was carried out by Alex and Mammen (2012) surveyed South African grade 10 learners' geometrical thinking level in view of the Van Hiele theory. A sample of 191 grade 10 learners from five senior secondary schools in one educational district in Eastern Cape in South African constituted the study participants. The study proved that the majority of the study participants gained and moved to Van Hiele level 3 from level 2.

Malasia, Abdullah and Zakaria (2013) also tried to improve students' level of geometric thinking through Van Hiele's phase-based learning. The quasi-experimental study that lasted six weeks involved 94 study participants: 47 study participants were in the treatment group, and 47 study participants were in the control group. The majority findings from this study showed that there was a statistically significant difference between the final levels of geometrical thinking between the control and the treatment group.

The study carried out by Abuand and Zaid (2013) sought to improve the levels of geometric thinking of secondary school students using a geometry learning video called Pembelajaran geometry, and was based on the Van Hiele theory. The video was shown to 150 students on different Van Hiele levels. 90 were on level 0, 60 were on level 1, and 30 were on level 2. The findings of this study indicated that the

majority of the study participants had shown an improvement on their geometric thinking level.

2.2.2 The use of Van Hiele models to improve the understanding of geometric concepts.

From the literature review conducted, the researcher thinks that not much focus is placed on the application of the Van Hiele theory in improving concept formation in geometry, but selected studies are presented here. Kotze (2007) concern was to use the Van Hiele theory to improve the knowledge base of a group of teachers and learners on the concepts of space and shape. Problems experienced in the concept formation in geometry were investigated and analysed. The paper concluded that the concept of shapes and space were problematic to both teachers and their learners.

In another study by Atebe (2008), Nigerian and South African students' conceptual understanding of triangles and quadrilaterals were investigated among 36 mathematics learners. The method of investigation involved identifying and naming shapes, sorting of shapes, stating the properties of shapes, defining shapes and establishing class inclusions of shapes.

Among the findings, Atebe found that the majority of the study participants were on Van Hiele level 0. Susan Connolly (2010) developed Regents units consisting of quasi-laterals based on the theories and instructional techniques of the Van Hiele model for 43 students who were enrolled in the high school. The study participants received instructions for using the newly developed material. The findings showed that the marks obtained by the study participants were better than the marks obtained by previous students who were taught without the new material.

2.2.3 Application of the Van Hiele instructional model in the classroom.

Sonja van Putten (2008) carried out a study which examined how geometrical concepts and knowledge were taught to a group of pre-service student teachers, using the Van Hiele theory of levels of teaching as the theoretical framework. The study found that many of the students' teachers were taught geometry with rote learning methods, using textbooks to present theorems and proof. Most of the proof exercises were obvious and were not challenging enough to force students to think while solving them. The students' teachers had forgotten geometric concepts which they had learnt immediately after matric and were not able to apply the concepts learnt in other situations. This implies that the ability to reason deductively was soon forgotten.

In another study, Tamer Kutluca (2013) investigated the impact of presenting geometric concepts to grade 11 learners by blending the use of 'Geogebra' (a teaching and learning software) and the Van Hiele instructional model. The quasi-experimental pre-/post-test control group design involved 42 study participants during the 2011 to 2012 spring term. The Van Hiele level of geometric understanding test developed by Usiskin (1982) and translated into Turkish by Duatepe, was used to collect the data. The study found that students in the experimental group were able to create their own geometric shapes and try different things with different shapes. They had the opportunity to participate actively in the instructional process. This implied they understood the concepts taught.

De Villiers (2003) designed instructional activities according to the instructive approach that were in accordance with the Van Hiele theory of learning geometry. The activities were designed to use a sketchpad to develop teachers' understanding of other functions of proof than just the traditional function of 'verification'. A solid theoretical rationale was provided for dealing with these other functions in teaching by analysing actual mathematical practice, where verification is not always the most important function. In view of the foregoing, one may say that the application of the Van Hiele learning instructional model are effective in systematically transferring the

concepts in geometry from the teacher and also developing the learners to begin to create their own knowledge of geometry.

Vojkurkova (2012) remarks that the Van Hiele theories should be transferred to other areas of mathematics such as algebra, functions, analysis, calculus etc. However, French (2004) linked students' ability in mathematics to their level of geometric conceptual ability. Furthermore, as successful as Van Hiele theories are in the teaching and learning of geometry, the majority of the work is done through the instructional approach. While geometric knowledge development and construction using the Van Hiele study, aims to empower learners to use the Van Hiele theories only as background information. Hence, this study aims to apply the Van Hiele theories to develop the learners' ability in the concept of geometric congruency in triangles.

Reeves and Muller (2005) cited recent research which contains evidence that there is a high level of under achievement in South Africa, particularly amongst learners at schools in high poverty areas. Although, as McDonnell (1995) argues, students can only be held accountable for their academic performance to the extent that the community (broadly defined) has offered them the tools to master the content expected of them.

This study is underpinned by van Hiele learning theory explained in subsection 1.2.1. This theory will guide the application of the van Hiele instructional model in the teaching of congruent of triangles in this study.

CHAPTER THREE

This chapter discusses the research design, sampling, data collection procedures, and instrumentation and ethical issues.

3 Methodology

3.1 Research Design

This study followed a mixed method approach in which the researcher used both qualitative and quantitative approaches in collecting the data and in analyzing the data. The mixed method approach is considered appropriate for this study because the researcher wants to:

- (1) Ensure that the Van Hiele instructional model (independent variable) was followed in the experimental fields while the teacher in the control research field used the traditional lesson delivery method.
- (2) Be able to account for, as well as justify, any improvement or decline in the study participants' learning performance in the topic of geometry taught at the end of the intervention.
- (3) Validate the results from the qualitative data analysis with the results from the quantitative data analysis, thereby triangulating the results.

The diagram below illustrates how the mixed method was applied in this study:

Figure 2 Mixed method



The Qualitative Approach

The descriptive research design involved uses non-participant unscheduled classroom observations to collect data on the natural setting of the research fields. The classroom visits were made to ascertain that classes in the experimental schools were conducted in line with the Van Hiele instructional model and to collect the necessary data. The behaviour of the study participants was observed and described

without necessarily influencing it. The data obtained from the classroom observations forms the main data of this study.

The Quantitative Approach

The quantitative method involved the pre-test post-test matching control causal-comparative design. This design was chosen because the researcher wanted to establish a relationship between the Van Hiele instructional model (independent variable) and the study participants' learning trajectory (dependent variable). The table below illustrates the quantitative research design:

Table 3: Illustration of the Pre-test Post-test matching Control Causal-Comparative design

Experimental Group	O ₁	X	O ₂
Control Group	O ₁	----	O ₂

O₁: Pre-test

O₂: Post-test

X: Intervention

Table 3 indicates that the pre-test (O₁) was administered to both the control and experimental groups at the start of the investigation. The intervention programme (X) was organized only for the experimental group; this implies there was no intervention for the control group. Then, both groups took the post-test (O₂) at the end of the enquiry. It's also important to note at this junction that the items of O₁ and O₂ were the same, but with different item numbers.

The pre-test/post-test design allowed the researcher to gather data on the study participants' performance from a class test. A pre-test was administered in all the groups, including the control group, after which all the groups were taught 'congruent triangles' for one week. At the end of the intervention week, a post-test was administered (the post-test items were the same with the items in the pre-test but the items were shuffled around so that question items did not retain the item number).

3.2 Sampling

The population of the study is the grade 10 learners in Gauteng province high schools in South Africa, which contains 15 education districts. The 15 education districts were clustered into three according to geographical location. The clustering of the 15 education districts was done to ensure that in the application of random sampling, schools that are close to each other were not selected to avoid compromising the study and to ensure the generalization of the research results. From each cluster, a random sampling approach was used to select a school where the study will be carried out.

Moreover, in each of the participating schools, an intact group of grade 10 learners in mathematics formed the study participants for the research. Two schools were randomly chosen as experimental schools, while the third one automatically became the control group. There is therefore an experimental group A, an experimental group B and the control group. The study participants were 136 in total: experimental group A had 48, experimental group B had 60, and the control group had 28. For ethical reasons, no school's nor study participants' names shall be mentioned. The 'experimental group A, experimental group B and control group' nomenclature shall be used throughout this study.

3.3 Data Collection

There are two types of data that were collected: qualitative and quantitative.

3.3.1 Qualitative Data Collection

The data gleaned from the classroom observation was qualitative in nature. Classroom observations were carried out in the experimental groups and the control group. In the control group, observation took place on Monday the 14th and Friday the 18th of July 2014. In the experimental group A, observation took place on Tuesday the 15th and Wednesday the 16th of July 2014. Observation of the experimental group B took place on Wednesday the 16th Thursday the 17th of July 2014. All the classroom observations were video recorded and field notes were taken as well. Thereafter, post-tests were administered and the researcher collected and

marked all the scripts. The study participants' post-test scripts were also used as part of the data collected for the problem solving data analysis.

However before the beginning of the intervention, the two mathematics teachers in the experimental groups were introduced to van Hiele's learning theory and instructional model a week before the commencement of the intervention. The training of the two teachers by the researcher took place from Monday 7th to Wednesday 9th, 2014. In preparation for the intervention.

3.3.2 Quantitative Data Collection

The data gleaned from the pre- and post-test design was quantitative in nature. The pre-test was written in all the participating schools: the control school test was on Wednesday, 9 July 2014, while it was written at the experimental schools A and B on Thursday the 10th and Friday the 11th of July 2014 respectively. The pre-test was carried out in the week that proceeded the intervention week. The intervention was carried out for one week in both experimental group A and experimental group B. In both the experimental groups, congruent triangles concepts were taught using the Van Hiele instructional model, while the same concepts were taught in a normal traditional chalk-and-talk approach in the control school. The intervention started on Monday, 14 July 2014, and ended on Friday, 18 July 2014. The post-test was written by all the groups on Tuesday, 22 July 2014.

3.4 Instrumentation

In this study, the same classroom test was the instrument that was used to collect quantitative data during pre-tests and post-tests, while a video recorder and note pads were used during the qualitative data collection.

3.4.1 Classroom Observation Instrument

The instruments used to gather data during the classroom observation were the Research Field Profile (RFP) checklist (see Table 4), Van Hiele's Instructional Approach (VHIA) checklist, a video camera and a writing pad for field notes. The RFP was a set of items expected to be found in the school and the VHIA are taken from the instructional model listed on section 1.2.2. These instruments need not be

validated neither do they need a reliability check. It is expected that electricity shall be constantly supplied throughout the period of the study, however new video camera batteries were made available in case of power outages.

In both experimental group A and experimental group B, separate examination pads were used to take field notes in order not to mix the data and to avoid confusion.

3.4.1.1 *Checklist for Van Hiele Instructional Strategy*

The Van Hiele instructional model is described in subsection 1.2.2. These instructional steps were supposed to be implemented during the teaching of the classes in the experimental groups A and B. The checklist was used to check if the experimental fields complied with the steps taken when teaching according to the Van Hiele instructional approach. This was done by going through the recorded classroom observation data and marking the items in the checklist that was covered by the teacher.

3.4.2 Classroom Test Instrument

As mentioned earlier on, the Pre-test and Post-test used one and the same instrument (see appendices 7 and 8). At the end of the intervention, study participants were expected to be able to display Van Hiele's geometric knowledge level 3 (abstract or relational), thus geometry knowledge acquisitions as appropriate for grade level 10 learners. The pre- and post-test instrument was used to gather data on the study participants' level of conceptual knowledge of congruent triangles acquired during the intervention (Alias, 2005).

3.4.2.1 *Development of the Classroom Test Instrument*

The researcher developed the classroom test items (questions) for the pre- and post-tests (see appendices 7 and 8). As mentioned earlier on, the same instrument was used for the pre-test and post-test. The instrument was made of supply items (essay type questions). It was used to measure the study participants' performance after the intervention, including the control group.

The test is structured in line with the nature of the geometric understanding expected of grade 10 (Van Hiele's geometric knowledge level 3).

A learner on van Hiele's level 3 geometric conceptual knowledge should be able to:

1. Recognize shapes on the basis of appearance and properties
2. Form abstract definitions
3. Know necessary and sufficient conditions for a concept
4. Demonstrate sufficient knowledge about relationships between shapes

It was in view of the above, that the general structure of the test took into account the following (see appendix 7 and 8):

1. To join the diagonals of a given shape
2. To identify information implied by a figure and separate (by drawing) the figure into different shapes
3. To demonstrate an understanding of necessary and sufficient conditions that establishes a concept
4. To identify general strategies of relating different shapes

The table of specification given below was used in drawing the instrument.

Table 4: Table of specification

Topic: Concepts	Cognitive Emphasis			
Content	Visualization	Analytical	Relational	Total
1. Name and define	4	6	10	20
2. Describe the shape	4	6	10	20
3. Problem solving	4	6	10	20
Total (Cognitive Emphasis)	12	18	30	60

Table 4 above was used as a guide for the preparation of the instrument.

3.4.2.2 *Validity of Classroom Test Instrument*

Three mathematics teachers who are experts in the field of mathematics and are currently teaching in two different secondary schools in the North West province (a province that is not part of the research field) were asked to scrutinize the pre- and post-test instrument in view of the mathematics curriculum under the CAPS educational policy. They conducted two levels of validation on the instrument: face and content validation.

The face validation involved checking the appropriateness of the language and test structure dimensions of the instrument, while in the content validation, the experts evaluated the extent to which the instrument items adhered to Van Hiele's geometric concept knowledge level 3.

The experts used the 4-level scale of appropriateness the researcher had prepared and the face and content validation of the scale is rated as follows:

1 = not appropriate

2= fairly appropriate

3= appropriate

4= highly appropriate

The items that returned appropriateness as less than 3 in either face or content validation were either discarded or restructured.

In addition, the following factors were identified as factors that can affect internal validity of the study: nearness of the research fields and the making the teachers and the study participants aware that the content of the pre and post-test (the test instruments) were the same. Hence, the researcher ensure that the selected schools were from different clusters during the sampling and as for the test instrument, the researcher did not disclose to the teachers nor the study participants that the content of the pre and post-test (the test instruments) were the same.

3.4.2.3 Reliability of the Classroom Test Instrument

The reliability of the classroom test instrument was measured using the intra-scorer reliability method. The ability of the test instrument to give consistent cognitive measurement of the geometric knowledge content taught is the main focus here.

To achieve the above reliability test, the classroom test instrument was administered to one of the grade 11 classes in a secondary school (there were grade 11A - C in the school) in North-West province (a different province from where the study was conducted) to avoid compromising the study. The number of learners that participated in this exercise was 21. The pre-test and post-test instrument was administered twice (X_1 , X_2): X_2 was administered three weeks after X_1 was administered, to ensure that learners did not remember the content of the first test. In addition, the instrument was not released to the learners after writing the test,

they were also not informed that the test would be repeated. Correlation between the scores of X_1 and X_2 were measured. A reliability coefficient of 0.74 was obtained.

3.5. Ethical Issues

The researcher obtained ethical clearance from the University of South Africa to carry out this study. In addition to this, the Gauteng Department of Education granted their permission to carry out the study at the relevant high schools used.

Similarly, the parents were also informed of the content of the study and they gave their permission for their children to participate in the study (see appendix 3 to 5). For ethical purposes, the name of the school, the teachers, and the learners involved in this study will remain anonymous.

CHAPTER FOUR

DATA ANALYSIS AND PRESENTATION OF RESULTS

In this chapter, the techniques for data analysis and results are presented.

4.0 Data Analysis and Results

Both qualitative and quantitative data were collected in this study. The data analysis techniques for qualitative data collected is presented first, followed by the quantitative data analysis technique. The results are presented school by school.

4.1 DATA ANALYSIS STRATEGIES

As mentioned in subsection 3.1, this study followed a mixed methods approach: a descriptive research design that involved classroom observation and pre-test – post-test matching control causal-comparative design. Data from the classroom observation were analysed using qualitative data analysis techniques, while the data from the causal-comparative design were analysed through quantitative data analysis techniques.

4.1.1 Qualitative Data Analysis Strategy

4.1.1.1 *Classroom Data Analysis Strategy*

The classroom observation data provides the text evidence of how the van Hiele instructional approach (the intervention) was followed in the teaching of congruent triangles in the experimental groups (appendix 6 was used for this purpose), as well as the evidence of the pedagogical approach used in the control group. Data was gathered through video recording of the on-site events and the use of field notes for important events. Only the qualitative data gathered from the experimental groups were analysed.

The steps involved in the data analysis procedure were as follows: first, the recorded data on the video tapes were transcribed (the process of transcribing the data was repeated several times to ensure that all the important events were captured), secondly, data from the field notes (for example, facial expressions during the

lesson) were put together with the transcribed data. The transcribed and the field notes data were sorted, and the features that were prominent were coded. The coded features were blocked and emerging themes were noted and the emerging themes from each group were also compared to avoid duplication of themes.

The features that emerged centred around classroom organization, the pedagogical approach of the teacher in the teaching of concepts of congruent triangles, the responses of the study participants to the pedagogical approach, the cognitive level of the questions the teacher asked the study participants, the cognitive level of the questions the study participants asked, and if the study participants were able to answer the teacher's formative and summative questions correctly. Interaction among the study participants, interactions between the teacher and the study participants, as well as of the study participants' postures during the intervention were also observed. Emerging themes were noted, gathered and compared. The data from each group was analysed and reported separately.

4.1.1.2 Problem Solving Data Analysis Strategy

The test instrument items were structured to test the following:

- (i) Identify and join appropriate ends of a shape that forms a diagonal
- (ii) Identify and draw shapes
- (iii) Use the properties of the identified shapes to show equality of sides or angles
- (iv) Use logical reasoning that may lead learners to determine the congruence of triangles

For the purpose of data analysis, the researcher categorized the post-test items into category A, category B, and category C according to the Van Hiele geometric learning level. See table 5 below.

Table 5: Learning Concepts Categories of the Test Instrument Items

Category	Question Item	Van Hiele Level
A	(i) and (ii)	1
B	(iii)	2
C	(iv)	3

The question item numbers placed in column 2 of this table show the numbers given to sub-questions in the post-test instrument.

To analyze the study participants' problem solving approaches the researcher used:

- (i) The number of study participants that passed each question category
- (ii) Solution appraisal method.

4.1.2 Quantitative Data Analysis Strategies

The quantitative data collected were captured in Statistical Package for the Social Sciences (SPSS). Descriptive and inferential statistical analysis techniques were used to study descriptive attributes of the data and the performance of the study participants in the post-test.

4.1.2.1 Descriptive Data Analysis Strategies

Descriptive data analyses were used to compare the means, and to present the standard deviation and skewness of the performance measurements in both the control and intervention groups. Charts were used to describe the spread of the performance of the study participants in the pre- and post-test.

4.1.2.2 Inferential Data Analysis Strategies

Inferential statistical analyses were performed on the pre- and post-test scores of both the control and experimental groups. The inferential statistical analysis was performed with a t-test. Data analysis techniques were used to investigate the significance of the impact of the intervention on study participants' performance in the post-test compared to the pre-test, (Erdogan, and Çelebi, 2009) and to describe

the nature of the impact. The results of the data analysis and the tested hypothesis are presented school by school.

4.1.2.3 Effect Size Analysis Strategies

According to Sherri Jackson (2014), Cohen's effect size (ES) is the effectiveness of an intervention compared to some others. In this study, the effectiveness of the Van Hiele instructional approach is compared to that of the chalk-and-talk traditional teaching approach.

In this study, The Cohen's ES for t-test is calculated as follows:

$$ES = \frac{\bar{x}_{\text{exp}} - \bar{x}_{\text{control}}}{SD_{\text{pooled}}}$$

Where:

ES = Effect size

\bar{x}_{exp} = the mean of the scores in the experimental group

\bar{x}_{control} = the mean of the scores in the experimental group

SD_{pooled} = the standard deviation using the experimental and control group data

The SD_{pooled} is calculated as follows:

$$= \sqrt{\frac{(n_{\text{exp}} - 1)(SD_{\text{exp}})^2 + (n_{\text{control}} - 1)(SD_{\text{control}})^2}{n_{\text{exp}} + n_{\text{control}} - 2}}$$

Where:

n_{exp} = sample size for the experimental group

n_{control} = sample size for the control group

SD_{exp} = standard deviation for the experimental group

SD_{control} = standard deviation for the control group

4.2 PRESENTATION OF RESULTS

The teaching profile of each mathematics teacher and their respective schools (research field) are presented first. This is followed by the results according to the research questions for each group. There are two experimental groups (experimental group A and experimental group B) and a control group. In the experimental groups, intervention (teaching congruent triangles' concepts using the van Hiele instructional approach) took place. The intervention period was one week, because one week was scheduled in the curriculum by the Department of Basic Education for the teaching of congruent triangles. The pedagogical approach for the control group was a chalk-and-talk, traditional teaching approach. Classroom observations were conducted in all the groups, and two sets of data (qualitative and quantitative) were collected and analysed from each group.

The classroom observation was structured to collect data on the natural settings of the research field and to be able to gather data on the physical activities that might influence the results of the study during intervention. A video recorder and field notes were used to collect relevant data (see section 3.3.1). None of the classroom observations was pre-scheduled with the teacher, but the researcher did arrange how he would visit each research field (see subsection 3.3.1).

An observation checklist was used (see appendix 6), after transcribing from the video, to measure compliance with the intervention. Relevant events were also recorded using field notes. The transcribed data and the data collected from the field notes were coded and processed. Emerging themes in the two observations within a group were compared (Mapolelo, 2003; Malone, 1996).

The results will be presented below. The results of the data analysis of the experimental groups shall be presented first, followed by the results of the data analysis of the control group. In each group, the results of the qualitative data analysis shall be presented first.

4.2.1 Summary of the Profile of the Research Fields

The researcher believes that the knowledge of each research field context may also be valuable in interpreting some of the results from the data analysis. Hence, table 6 below provides a summary of the profile of the research fields.

Table 6: Research Fields Profile

ITEMS	CONTROL	EXPERIMENT GROUP A	EXPERIMENT GROUP B
THE SCHOOL			
- The student population in the school.	1158	560	1316
- The total number of classrooms	33	15	26 (with 12 additional mobile classrooms)
- Total number of teachers			
- Total number of maths teachers	45	26	46
- Average number of learners per class	9	7	8
- Classroom furniture			
- Technology teaching aid	35	37	50
- Nature of the library			
-			
STUDY PARTICIPANT TEACHERS	1 Learner per desk	2 Learners per desk	1 Learner per desk
	None	None, use chalkboard	None, use chalkboard
	No library	None	1 library room with few books. Used mainly for studies.

- Age	Late thirties	Forties	Twenties
- Qualification	B.Ed (Maths)	B.Ed (Mat's)	B.Ed (Math's)
- Teachers	7	8	9
- Experiences	6 years	23 years	3 years

4.3 Experimental Group A

4.3.1 Presentation of the results of the qualitative data analysis in experimental group A

How does the Van Hiele instructional model facilitate the learning of congruent triangle concepts in the participating high schools in the experimental schools?

The results from the analysis of the qualitative data suggest that the intervention might have facilitated the learning of the concepts of congruent triangles. Details of the results of the data analysis are presented below:

(i) Results of the Classroom Observation Data Analysis

In this group, the intervention (the use of Van Hiele's instructional model in the teaching of congruent triangles, section 1.2.2 of this study) was from 14 July to 18 July 2014. There were three grade 10 classes (Class 10A, Class 10B, and Class 10C). Classes 10A and B did mathematics, while Class 10C did mathematical literacy. Therefore, the study participants in this school consisted of the intact groups of grade 10A and B. There were 25 registered learners in Class 10A and 22 registered learners in Class 10B, which amounted to a total of 47 study participants.

The two classes were scheduled together on the mathematics time table. During the mathematics lessons, study participants sat in groups of about four. They placed together two or three lockers and sat around the lockers. Mathematics was scheduled for 35 minutes per period and it is always a double period. The learners received their mathematics tuition in the classroom which served as the permanent class of the teacher.

There were two different unscheduled classroom observations that were conducted (on 15 and 16 July 2014 respectively). The researcher observed the first lesson on congruent triangles in this group.

The results of the data analysis showed that the teacher taught the concept of congruent triangles according to the Van Hiele instructional model. In addition, it was revealed that the study participants yielded positively to the instructional strategy used to teach the concept of congruent triangles: they researched, tried and explored, and were responsive to the teacher's questions. The results also indicated that the study participants were asking questions, and also answering questions in a way that could suggest conceptual understanding of congruent triangles.

They were quiet when the teacher wanted to explain something to them, they always took a moment to think before responding to the teacher's questions and started to discuss when discussing in groups or when there was an open question from the teacher. Below is the unabridged excerpt of the first classroom observation in this group.

Teacher: *"Today we are starting a new topic ', who can describe a triangle?* He paces up and down in front of the class, as he observes who will answer the question. A male study participant indicates that he wants to try.

Study Participant: *"It is a three-sided figure."*

Teacher: *"Thank you, sit down."*

Teacher: *"Somebody to name different types of triangles that we have."*

Instantly low murmuring is heard. After about 3 minutes, two study participants from different parts of the class raise their hands. The teacher points to one of the study participants that has her hand in the air:

Teacher: *"You, yes."*

Study Participant: *"Right angled triangle... hm, hmm."* There is whispering from group members, then she continues: *"Isosceles trianglehm, hmm."*

Teacher: *"Yes, sit down. Who wants to assist her?"* The teacher continues: *"Now I want somebody from the front row. You."* He points at a lady sitting in one of the groups in the front row.

Study Participant: *"Scalene triangle, right angled triangle."*

Teacher: *"Good."* The teacher goes to the board and draws a diagram of two right angle triangles.

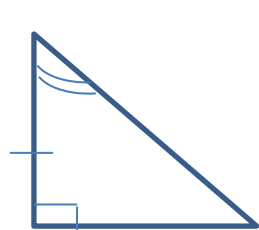


Diagram A

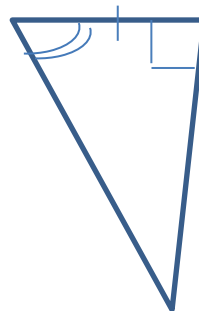


Diagram B

Teacher: *"Look at the two diagrams on the board."* Teacher points at diagrams A and B on the board. *"What types of triangles are these triangles?"*

Teacher: *"You, answer the question."* He points at a lady in the middle row.

Study Participant: *"Right angle triangle"*, answers the student.

Teacher – *"What are the similarities you can draw from these two diagrams?"*

According to the data analysis on results from the field note, it is here where the study participants start to look lost or confused. The teacher goes over the question again, but now looking at the study participants:

Teacher: *"What are the similarities you can draw from these two diagrams?"* He repeats the question.

Study Participant: *"I see that one side of the triangles are marked and there are two marks on each angle."*

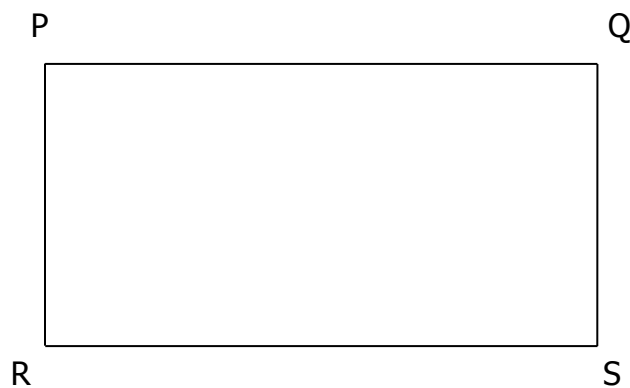
Teacher: *"Good, what is the implication of these."*

Study Participant: *"Maybe they are similar or equal"*, answers a student from one of the back rows.

The teacher goes to the white board with a blue white board marker in his hand and writes on the board *"Congruent Triangle, this is our new topic. If you look at the two triangles, the double angle sign on both diagrams implies that the angles are equal on both triangles"*. He pauses for a few seconds and then continues: *"In the same way, one stroke on the sides of the diagrams implies that the sides are equal to each other"*

The teacher writes the following question on the board:

Given that PQRS is a rectangle.



Use the properties of rectangles to show that $|PR| = |QS|$, if $|PQ| = |RS|$ and are parallel to each other.

The teacher goes around to see how the study participants are thinking and trying to solve the problem. After about 7 minutes, the teacher asks one of the students to show his attempts.

Study Participant: *"Opposite sides of rectangles are equal and side PR is opposite to side QS, I think $|PR| = |QS|$."*

Teacher: *"Clap for him."*

The teacher goes to the board, pointing at the triangles drawn before: *"We shall use these triangles to explain congruency of triangles."*

Teacher: *"How can you explain what it means when two triangles are congruent to each other?"*

There is a moment of silence, then one of the study participants answers:

Study Participant: *"Two triangles drawn beside each other."*

Teacher: *"Yes, any other answer? Think, discuss with the person next to you."*

There is murmuring and two of the study participants, each from different parts of the class, who are sitting alone go to join other groups.

Teacher: *"Two triangles drawn beside each other and there are similarities in terms of the sides or angles between the two triangles"*, the teacher explains. He asks another question: *"Mention conditions needed for congruency in triangles?"*

This is to get the study participants thinking. The way the study participants sit in groups might have help them to discuss among themselves at this stage. Nobody responds. He then states all the conditions needed for congruency between two triangles as follows:

Teacher: *"Consider these two triangles."* He points at the two triangles he on drew on the board earlier, which he labelled ABC and DEF, and continues to write:

- (i) If the three sides of $\triangle ABC$ are equal to the three sides of $\triangle DEF$, this condition is written as *SSS*.

The teacher touches the respective sides of the two triangles, to show the sides he is referring to. He then continues to write:

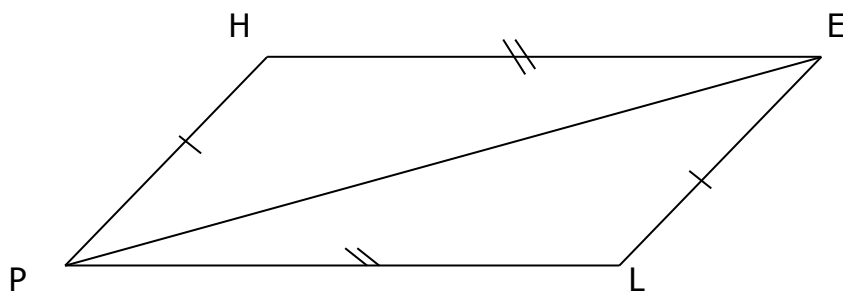
- (ii) If two sides of $\triangle ABC$ are equal to two sides of the $\triangle DEF$ and the angle included by those two pairs of equal sides are equal, this condition is written as *SAS*.

As the teacher is writing, he touches the specific places on the diagrams he is referring to.

- (iii) If two angles of $\triangle ABC$ are equal to two angles of the $\triangle DEF$ and any one side of $\triangle ABC$ is equal to the corresponding side of $\triangle DEF$. This condition is written as *SAA*.
- (iv) If two right angled triangles have their hypotenuses equal and also one other side of the triangle equal to one other side of the other triangle. This condition is written as *RHS*.

He goes to board to write the following example:

If PHEL is a parallelogram, show that $\triangle PHE$ is congruent to $\triangle PLE$ in the diagram below:

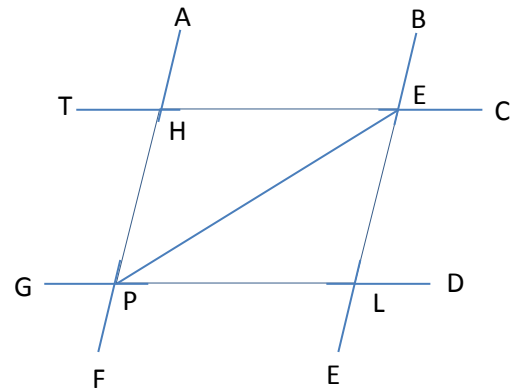
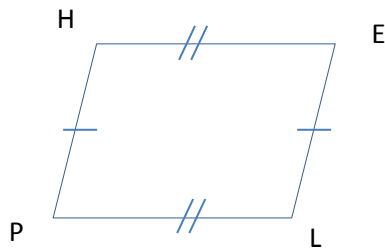


Teacher: *"How can we do this? Anybody?"*

The study participants think for a while, the classroom is quiet, they start raising their hands.

Teacher: *"Yes, you."* He points to a boy at the back. *"Come and help us with this."*

A boy comes and presents the following argument:



The study participants explain this problem by extending the sides of the parallelogram as shown above.

Study participant:

"Since $\widehat{AHT} = \widehat{PLE}$...alternate angles

$\widehat{AHT} = \widehat{PHE}$...opposite angles

$$\therefore \widehat{PHE} = \widehat{PLE}$$

Since $\overline{PH} = \overline{LE}$...shown on the board

and $\overline{HE} = \overline{PL}$...shown on the board

Then we have SSA condition."

Another study participant raises her hand.

Teacher: *"Yes, do you want to contribute to what he did or disagree with it?"*

Study Participant: *"I think we can solve it by..."*

Teacher: *"No go and show us your own thinking on the board."*

She goes to the board.

Study Participant: *"Since line PE is common, and line HE is equal to line PL and line PH is equal to line LE, then we have SSS and $\widehat{PHE} = \widehat{PLE}$."*

Teacher: "Good."

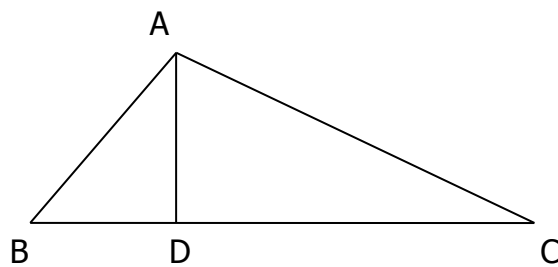
By this time he checks his watch and he announces:

Teacher: "We only have fifteen minutes left, I will give you some exercises to do in your workbook."

He writes the following exercises on the board:

EXPERIMENT A.

1.

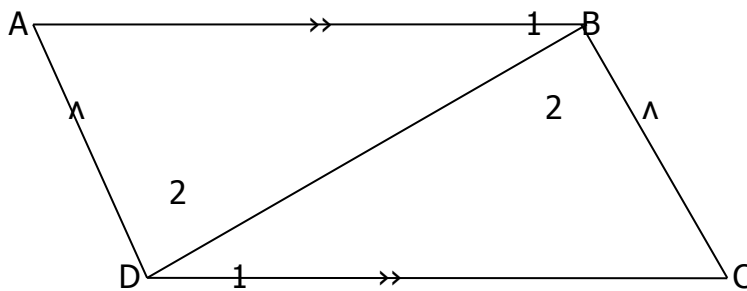


ABC is a right-angled triangle with $\hat{A} = 90^\circ$.

$AD \perp BC$

Prove that $\triangle BAD \cong \triangle ACD$

2. ABCD is a parallelogram



(a). Show that:

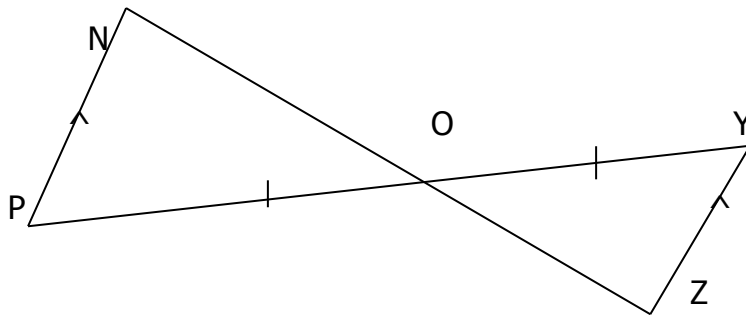
(b). $\hat{BAD} = \hat{BCD}$

(c). $\hat{ABD} = \hat{BDC}$

Hence, Prove that $\triangle ABD \cong \triangle CDB$.

QUESTION 3

(a). Prove that $\triangle NOP \equiv \triangle YOZ$



The teacher goes around as the study participants are solving the exercises to guide them.

After this, the teacher summarizes the concept of congruent triangles to the study participants. This marks the end of the lesson.

(ii) Results of the Data Analysis on Problem Solving Approach

(a) The number of study participants that passed each category

Table 7: Number of Study Participants that Pass each Question Category

Category	Question Items that falls into each Category	Number of the study participants that scored 50% and above
A	(i) and (ii)	41 (86%)
B	(iii)	33 (69%)
C	(iv)	27 (56%)

Table 7 shows that 41 (86%), 33 (69%) and 63 (59%) of the study participants scored 50% and above in the question category A, B and C respectively.

(b) Solution Appraisal Data Analysis Results

The solution appraisal results are presented based on the question category.

Question Category A

Sample question is: *"Join the necessary points in the figure above to form diagonal /QR/."*

The results of the solution appraisal data analysis show that the majority of the study participants did not have a problem with locating the two corners of the shape given to be connected to form the required diagonal in question category A (see table 7 above). In addition, it also shows that the majority of these study participants understand the term 'diagonal'. This confirms that majority of the study participants are comfortable on van Hiele level 1

Question Category B

Sample question is: *"Identify and draw $\triangle PRS$ and $\triangle QRS$ separately."*

Again the results of the solution appraisal data analysis shows that the majority of the study participants did not have a problem with identifying the necessary triangles from the original figure, hence drawing the triangles was not difficult for them. In addition, it also shows that the majority of these study participants understood the term 'triangle' and could trace a triangle out of a given figure. This confirms that majority of the study participants are comfortable on van Hiele level 2

Question Category C

Sample question is: *"If an $\hat{PQS} = \hat{RQT}$ and $\hat{QPR} = \hat{QST}$, prove that $\hat{QRP} = \hat{QTS}$."*

or *"Hence prove that $\triangle QPR \equiv \triangle QST$."*

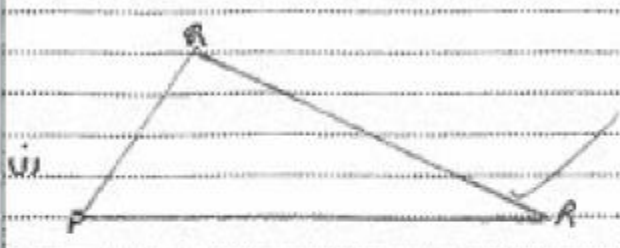
In this question category, the analysis shows that more study participants in this group struggled to approach the question items correctly compared to the question categories A and B, as the questions demand a higher cognitive input. Actually none of the study participants got the question *"If an $\hat{PQS} = \hat{RQT}$ and $\hat{QPR} = \hat{QST}$, prove that $\hat{QRP} = \hat{QTS}$ "* perfectly correct. However, some made good attempts at solving it.

But the study participants approached the questions to prove the congruency of triangles with more clarity and they were able to apply congruent triangle concepts compared to the other question in this category. This confirms that though some of the study participants attained van Hiele level 3, but many may still more time with their studies to be on the van Hiele level 3.


Figure 3 below is a sample script that shows how the study participants approached the post-test questions:

Figure 3 Sample script that shows how participants approached the post-test questions

Question 1



$\frac{10}{18}$



$\frac{2}{2}$

In $\triangle PQR$

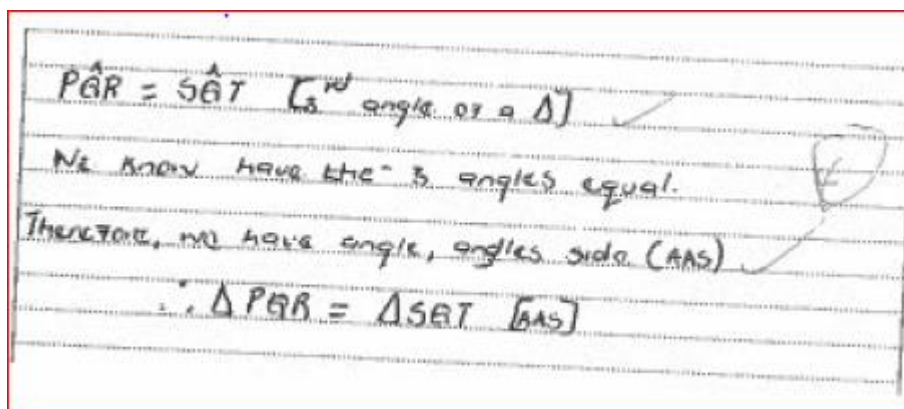
$$\angle P + \angle Q + \angle R = 180^\circ$$

[Sum of \angle s of \triangle]

In $\triangle SAT$

$$\angle S + \angle A + \angle T = 180^\circ$$

$\frac{14}{14}$



The sample script placed in figure 3 above is the solution approach of one of the study participants to question 1 in the post-test. There were three sub-questions: i, ii, and iii. The sub-question (i) was in the A question category (see 4.1.1.2) of the post-test (see appendix 7). The script shows that the study participant who wrote this answer could easily identify and draw the specified triangle.

However, the study participant managed to attempt sub-question (ii), which belongs to question category C, by stating the sum of angles in a triangle, which was a good attempt. For sub-question (iii) which is a question category C, the study participant was able to assume that $\angle QRP = \angle QTS$ and was therefore able to solve the sub-question (iii) correctly.

4.3.2 Results of the analysis of the quantitative data analysis in experimental group A.

How does the Van Hiele instructional model impact on study participants' score achievement in the learning of the concepts of congruent triangles in the experimental schools?

The results emanating from the quantitative data analysis indicated that the study participants in the experimental groups performed better than the study participants

in the control group in the post-test, although all the study participants performed poorly in the pre-test. The results from the data analysis are presented below:

The descriptive, statistical results below were indicative of the improved achievement in the learning of the concepts of congruent triangles. See the results below:

(i). Results of the descriptive data analysis

Table 8: Descriptive Results in Experimental group A

	N	Mean	Std. Deviation	Std. Error Mean (SEM)
Pre-test	48	15.5833	6.02065	.86901
Post-test	48	61.2500	17.99113	2.59680

In experimental group A, table 8 shows that the mean score of the pre-test is 15.58%, while that of the post-test is 61.25% (rounded up to 2 decimal places). The standard deviations are 6.02% and 17.99% for the pre-test and post-test, respectively. This implies that in the pre-test, the majority of the scores result fell within the (9% - 21%) category, while in the post-test, the majority of the scores result fell within the (44% - 78%) category. In addition, the high standard deviation of 17.99% showed that the scores were widely spread within the indicated scores category.

The standard deviation error of 0.87% and 2.60% (to 2 decimal places) reflect the degree of accuracy as the mean for the data collected.

The above results are further presented graphically. This presentation of the results allows one to see the spread of the scores both in the pre-test and post-test.

Figure 4 The Histogram of the Pre-test Results in Experimental group A

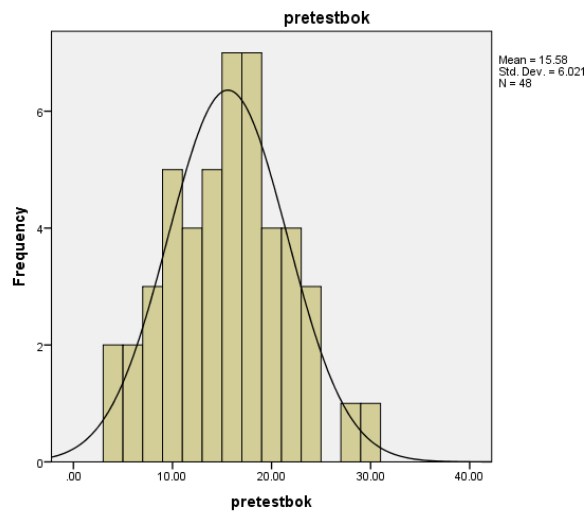


Figure 4 is a pictorial view of the results in table 8. In figure 4, it is evident that the majority of the score results fell within the (9% - 21%) category, as categorised in table 8. This figure is also a normal distribution, which indicates that as the curve approaches zero on both the right and left sides of the graph, the limits are about 3% and 31% respectively.

Figure 5: The Histogram of the Post-test Results in Experimental group A

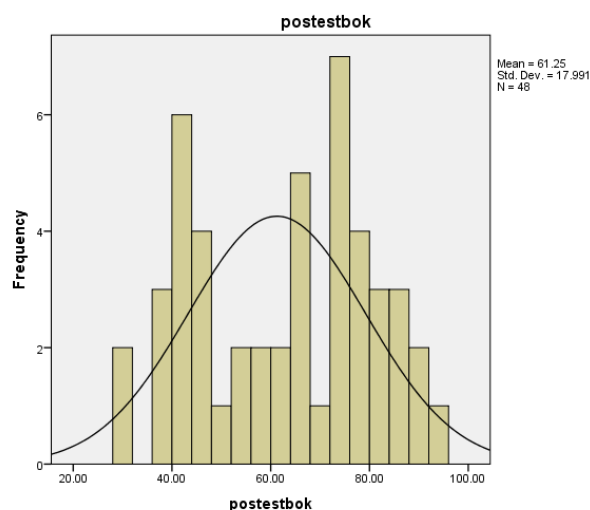


Figure 5 visually illustrates the results in table 8. In figure 5, the researcher observed that the majority of the score results spread across the (44% - 78%) category as categorised in table 8. The graph is also a normal distribution, which indicates that as the curve approaches zero on both right and left sides of the graph, the limits are about 10% and 96% respectively.

Although the descriptive statistics show the improved achievement in the post-test over that of the pre-test, the results of the inferential data analysis below shows the statistically significant comparison of the two means.

(ii) Results of the inferential data analysis

Table 9 One - Sample Test

	Test Value = 50					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Pre-test Group A	-39.605	47	.000	-34.41667	-36.1649	-32.6685
Post-test Group A	4.332	47	.000	11.25000	6.0259	16.4741

Table 9 provides the results of the t-test comparison of the means of both the pre-test and the post-test. It is revealed that there is a statistically significant difference between the two means, since $F < 0.005$. From table 8 above, it is clear that the means for both the pre- and post-test were about 15 and 61 respectively; this implies that there is a statistically significant improvement in the scores of the study participants in this group.

Effect Size Result

$$SD_{\text{pooled}} = \sqrt{\frac{(48-1)(18)^2 + (28-1)(8.2)^2}{48+28-2}}$$

$$= \sqrt{\frac{17043.48}{74}} = \sqrt{230.32} = 15.18$$

and

$$ES = \frac{61.25 - 26.36}{15.18} = 2.3$$

For full effect size calculation, please see appendix 11

4.4 Experimental Group B

4.4.1 Presentation of the results of the qualitative data analysis in Experimental group B

How does the Van Hiele instructional model facilitate the learning of congruent triangle concepts in the participating high schools in the experimental schools?

The results gleaned from the qualitative data analysis suggested that the intervention might have facilitated the learning of the concepts of congruent triangles. The results of the data analysis are presented below.

(i) Results of the Classroom Observation data Analysis

The structure of the classroom observation data collection and analysis in this group was similar to that of the experimental group A, and in order to avoid unnecessary repetition the full classroom observation will not be relayed, but the results will be presented.

In this group, the intervention (the use of the Van Hiele instructional model in the teaching of congruent triangles, section 1.2.2 of this study) was from 14 to 18 July 2014. There were four grade 10 classes (Class 10A, Class 10B, Class 10C, 10D). Classes 10A and B did mathematics, while Classes 10C and D did mathematical literacy. Therefore in this school, the study participants are the intact group of grade 10A and B. There were 32 registered learners in Class 10A and 28 registered learners in Class 10B, which added up to 60 learners in experimental group B.

These two classes were scheduled together on the mathematics time table. During the mathematics lessons, study participants sat around two or three lockers in groups of about four, exactly as the students in research field A did. There were six mathematics periods in a week: on Mondays and Wednesdays a single period, double periods on Tuesdays and Thursdays, and no class on Friday. Each period was 35 minutes. The learners attended the mathematics class in the class which served as the permanent class for the teacher, as was found in research field A. There were two different unscheduled classroom observations that were conducted.

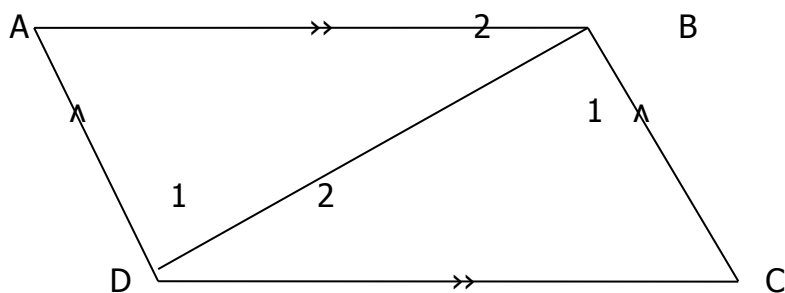
The classroom observation took place on Wednesday the 16th, and Thursday the 17th of July 2014. The Wednesday class was the first lesson on congruent triangles in this group; the researcher was about 15 minutes late to this lesson since he had to rush down to this research field from the experimental group A class. However, the teacher was still at the ice breaking stage of the lesson when the researcher got into the class. It was a single period lesson.

The results of the data analysis showed that the teacher followed the Van Hiele instructional steps in teaching the concept of congruent triangles. The study participants responded positively to the intervention, even though it was different from how the teacher initially taught them. This reflected in the field note data as comments made by one of the study participants showed that they responded very well throughout the intervention. It was revealed that the teacher was not giving the study participants enough time to think when he asked questions, and the teacher was not digging deep enough into the study participants' prior knowledge, which could have helped them to reason properly.

The results indicated that the study participants were indeed able to answer questions and ask intelligent questions themselves. It also emerged from the data analysis that the type of questions many of the study participants were asking could suggest conceptual understanding of congruent triangles. But the facial expressions of a few of the study participants might have indicated that some of them could not cope with the approach of allowing learners to reason out their answers, specifically when the task is of such high cognitive demand.

Below are the exercises the teacher gave in the second classroom observation as a sample of exercise questions discussed in the class.

Question 1. ABCD is a parallelogram

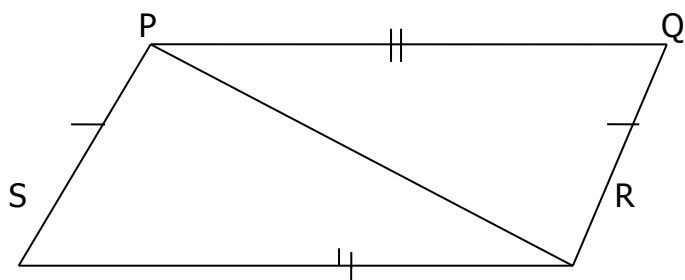


(a). $\hat{A} = \hat{C}$

(b) $\angle B = \angle D$

(c). Prove that $\triangle ABC \equiv \triangle CDA$

Question 2. PQRS is a parallelogram with $PQ = RS$ and $PS = QR$.



Prove that

(a). $\triangle PQR \equiv \triangle RSP$.

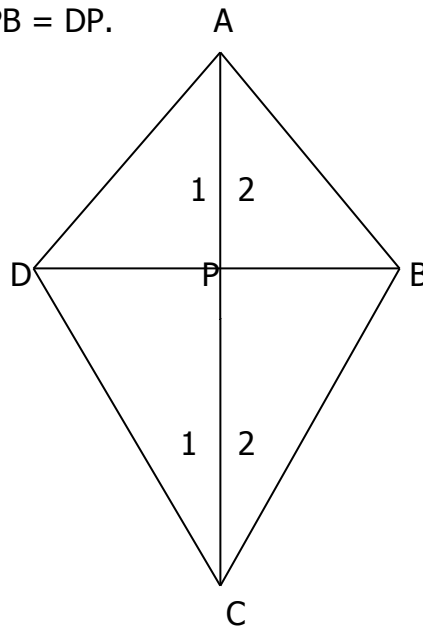
(b). $PQ \parallel RS$ and $PS \parallel QR$.

c). PQRS is a parallelogram.

Question 3.

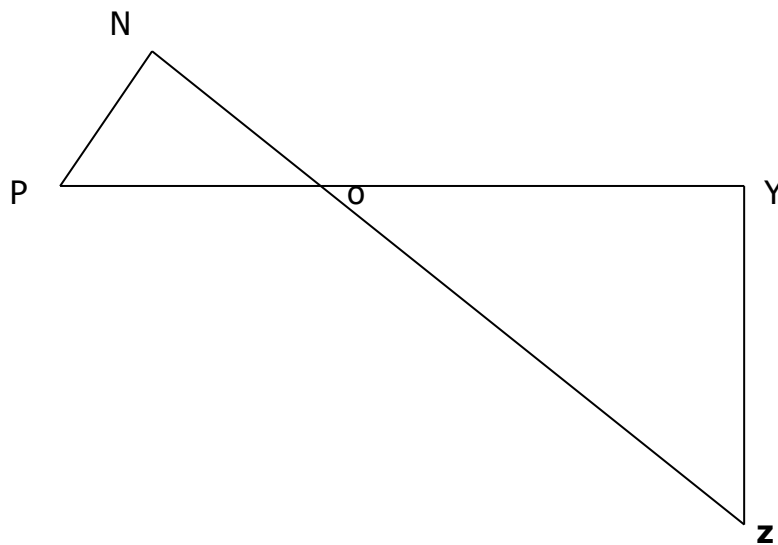
ABCD is a kite with diagonals AC and BD intersecting at P, and $AC \perp BD$.

Also $AD = AB$, $CD = BC$ and $PB = DP$.



Question 4.

(a). Prove that $\triangle NOP \equiv \triangle YOZ$.



(ii) Results of the Data Analysis on Problem Solving Approach

(a) The number of study participants that pass each category

Table 10: Study Participants that Pass each Question Category

Category	Question Items that falls into each Category	Number of Study Participants that scored above 50%
A	(i) and (ii)	53 (84%)
B	(iii)	35 (59%)
C	(iv)	31 (52%)

Table 10 shows that 53 (84%), 35 (59%) and 31 (52%) of the study participants scored 50% and above in the question categories A, B and C respectively.

(b) Solution Appraisal Data Analysis Results

The researcher presented the solution appraisal results based on the question category.

Question Category A

Sample question: *"Join the necessary points in the figure above to form diagonal /QR/."*

The outcome of the solution appraisal data analysis was similar to the results obtained in group A. The solution appraisal data analysis showed that the majority of the study participants did not have a problem locating the two corners of the shape given to be connected to form the required diagonal in question category A (see table 8 above). This confirms that majority of the study participants are comfortable on van Hiele level 1.

Question Category B

Sample question: *"Identify and draw $\triangle PRS$ and $\triangle QRS$ separately."*

Again the results of the solution appraisal data analysis showed that the majority of the study participants did not have a problem with identifying the necessary triangles from the original figure and hence drawing the triangles was not difficult for them. In addition, it also showed that the majority of these study participants

understood the term 'triangle' and could trace a given triangle out of a given figure. This confirms that majority of the study participants are comfortable on van Hiele level 2

Question Category C

Sample question: "If an $\hat{PQS} = \hat{RQT}$ and $\hat{QPR} = \hat{QST}$, prove that $\hat{QRP} = \hat{QTS}$ "

or "Hence prove that $\Delta QPR \equiv \Delta QST$."

In this question category, the analysis showed that more study participants in this group struggled to approach the question items in this category correctly compared to the question categories A and B. Actually none of the study participants got the question "If an $\hat{PQS} = \hat{RQT}$ and $\hat{QPR} = \hat{QST}$, prove that $\hat{QRP} = \hat{QTS}$ " perfectly correct, however, some made good attempts at solving it. The study participants approached the questions on the concepts of congruent of triangles with more clarity and they were able to apply congruent triangles' concepts compared to the other question in this category. This confirms that though some of the study participants attained van Hiele level 3, but many may still more time with their studies to be on the van Hiele level 3.

4.4.2 Results of the analysis of the quantitative data analysis in experimental group B

How does the Van Hiele instructional model impact on study participants' score achievement in the learning of the concepts of congruent triangles in the experimental schools?

The results emanating from the quantitative data analysis indicated that the study participants in the experimental groups performed better than the study participants in the control group in the post-test, though all the study participants performed poorly in the pre-test. The results from the data analysis are presented below:

The descriptive statistics results below were indicative of the improved achievement in the learning of the concepts of congruent triangles. See the results below:

(i) Results of the descriptive data analysis

Table 11: Descriptive Results in Experimental group B

	N	Mean	Std. Deviation	Std. Error Mean
Pre-test	60	13.6333	6.19996	.80041
Post-test	60	41.2333	7.24479	.93530

In experimental group B, table 11 shows that the mean score of the pre-test is 13.63%, while that of the post-test is 41.23% (rounded up to 2 decimal places). The standard deviations are 6.02% and 7.24% for the pre-test and post-tests respectively. This implies that in the pre-test, the majority of the score results fell within the (8% - 20%) category, while in the post-test, the majority of the scores fell within the (34% - 48%) category. The standard deviation error of 0.80 and 0.94 (to 2 decimal places) reflect the degree of the accuracy of the mean of the data collected.

The above results are further presented graphically, which enables one to see the spread of the scores both in the pre-test and post-test.

Figure 6 The Histogram of the Pre-test Results in Experimental group B

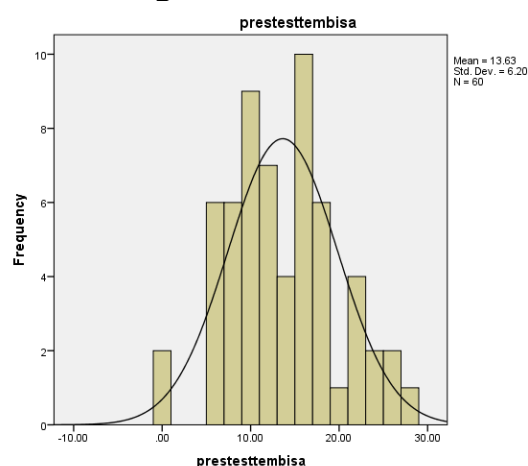


Figure 6 gives a pictorial view of the results in table 11. It can be observed that the majority of the score results fell within the 8% and 20% category as analysed in

table 11. The graph is also a normal distribution, this indicates that as the curve approaches zero on both the right and left sides of the graph, the limits are about 0% and 28% respectively.

Figure 7 The Histogram of the Post-test Results in Experimental group B

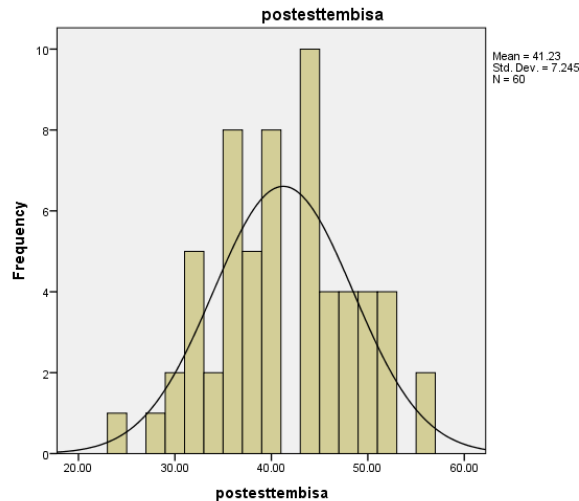


Figure 7 provides a pictorial view of the results in table 11. It is observed here that the majority of the score results spread across the 34% and 48% category, as analysed in table 11. The figure is also a normal distribution; this indicates that as the curve approaches zero on both the right and left sides of the graph, the limits are about 24% and 55% respectively.

Although the descriptive statistics show an improved achievement in the post-test over that of the pre-test, the results of the inferential data analysis below show the statistical significant comparison of the two means.

(ii) Results of the inferential data analysis

Table 12: Inferential Data Analysis Results

::	Test Value = 50					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Pre-test						
Group B	-45.435	59	.000	-36.36667	-37.9683	-34.7650
Post-test						
Group B	-9.373	59	.000	-8.76667	-10.6382	-6.8951

Table 12 shows the result of the ANOVA comparison of the means of both the pre-test and post-test. It is revealed that there is a statistically significant difference between the two means, since $F < 0.005$.

Effect Size Result

$$SD_{\text{pooled}} = \sqrt{\frac{(60-1)(7.2)^2 + (28-1)(8.2)^2}{60+28-2}}$$

$$= \sqrt{\frac{4874.04}{86}} = \sqrt{56.67} = 7.53$$

and

$$ES = \frac{41.23 - 26.36}{7.53} = 1.97$$

For full effect size calculation, please see appendix 10.

4.5 Control Group

4.5.1 Presentation of the results of the qualitative data analysis

How does the Van Hiele instructional model facilitate the learning of congruent triangle concepts in the participating high schools in the experimental schools?

The reader should note that there is no intervention in this group as the teacher in this group used the traditional instructional approach.

However, the results that emanated from the analysis of the qualitative data suggested that the traditional instructional approach used in this group might not have facilitated the learning of the concepts of congruent triangles. The results are further presented below.

(i) Results of the Classroom Observation Data Analysis

In this group, the study took place from 9 to 18 July 2014. There were three grade 10 classes (Class 10A, Class 10B, Class 10C). It was only grade 10A that did mathematics, while Classes 10B and C did mathematical literacy. In this school, the study participants were the intact group of grade 10A. There were 28 registered learners in Class 10A, and the study participants in this control therefore numbered 28. During the mathematics lessons, study participants were seated one learner per locker. There were five mathematics periods in a week: on Mondays a single period, Tuesdays and Thursdays double periods, and no mathematics class on Friday.

Each period was 35 minutes. The learners have to go for mathematics class in the classroom which serves as the permanent classroom for the teacher. There was only one unscheduled classroom observation that was conducted (Monday 14th, 2014). The Monday class was the first lesson on congruent triangles in this group, the researcher was in the class before the class started. When it was time to start the lesson, the teacher went to the front of the class and started the lesson.

The reader should remember that no intervention took place in the control group. The teacher in the control group taught triangle congruency as he normally teaches using the textbooks, chalk and blackboard, leading the learners through the learning

of the concept of congruent triangles. The duration of the mathematics lesson in this group was 35 minutes.

There were two classroom observations, the results of the data analysis show that the classroom pedagogy followed the traditional teaching approach. The results indicated that the teacher normally writes the topic of the day on the board; explains the underpinning concept using some examples, and gives an illustration followed by formative exercises.

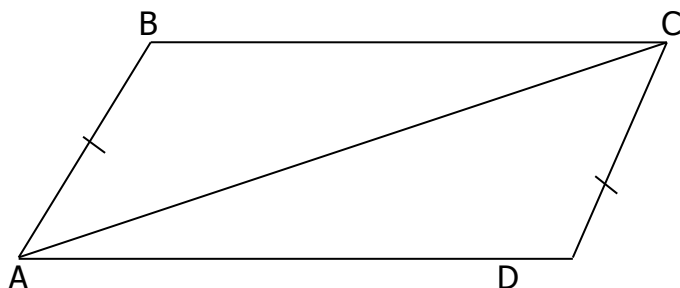
Below is an unabridged presentation of the classroom presentation observed:

The teacher went to the blackboard and wrote 'Congruent Triangles'

Teacher: "Our topic for today is as written on the board. Now, I want everybody to read out."

The study participants chorused the topic as written on the board.

He then drew the diagram below on the board.



Teacher: "What is the name of the shape on the blackboard?"

The study participants keep quiet.

Teacher: "Do you want to tell me, you do not know what a parallelogram looks like. Well that is a parallelogram ABCD."

He pauses a little, and then paces up and down at the front of the class.

Teacher: "The diagonal line AC divides the diagram into two triangles. Congruent triangles are triangles that are similar in all respect, sides, shapes and angles." He looks at the board, and goes to the board.

He uses the chalk in his hand to trace out the $\triangle ABC$ and $\triangle ADC$ that make up the parallelogram ABCD.

Teacher: *"Do you see these triangles?"*

Study participants: *"Yes."*

Teacher: *"I will use them to explain the congruent in triangles."*

He scans through the whole class with his eyes and goes to his table to fetch the mathematics textbook.

Teacher: *"Open your mathematics textbook on page 202, the conditions for congruency in triangles are there."*

He then goes to the board to copy what is in the book onto the board.

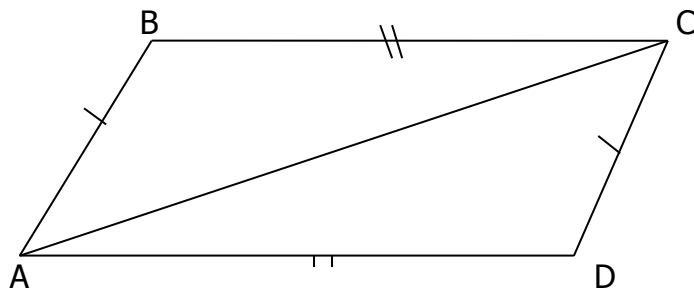
Teacher: *"The conditions on page 202 are as follows: Two triangles are congruent if:"*

Conditions of Congruency	Symbol (Notation)
<ul style="list-style-type: none">Three sides of one triangle are equal in length to the three sides of the other triangle.	SSS
<ul style="list-style-type: none">Two sides and the included angle in one triangle are equal to two sides and the included angles in the other triangle are equal to two angles and the sides of the other triangle.	SAS
<ul style="list-style-type: none">Two angles and one side of one triangle are equal to the corresponding two angles and sides of the other triangle.	AAS
<ul style="list-style-type: none">In two right-angled triangles, the hypotenuse and a side of one triangle are equal to the hypotenuse and a side of the other triangles.	RHS

Study participants: *"Are we to measure the sides to be sure it is the same?"*

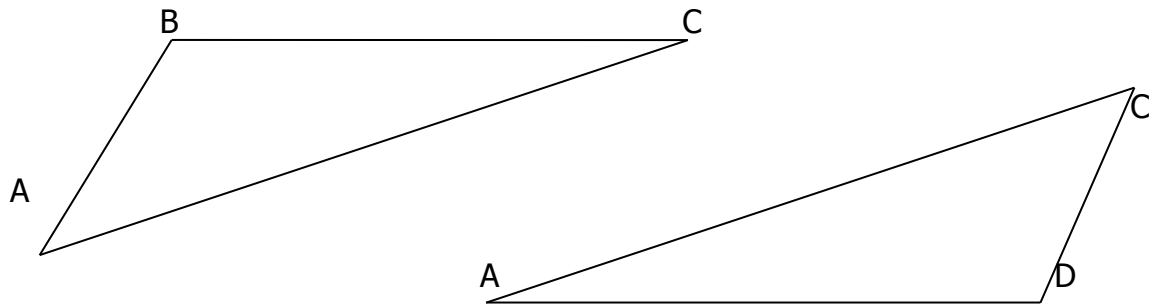
Teacher: *"No, it will be given to you."*

He then uses the diagrams of the parallelogram he has drawn on the board to formulate an example. He makes the following additions to the diagrams:



Teacher: *"In the diagram, there are two triangles $\triangle ABC$ and $\triangle ADC$."*

He splits the triangles as follows:



Teacher: I = k	opposite sides of parallelogram ABCD
J = L	opposite sides of parallelogram ABCD
M = N	diagonals of parallelogram ABCD
B = D	opposite angles of parallelogram ABCD

He turns to the study participants in the classroom.

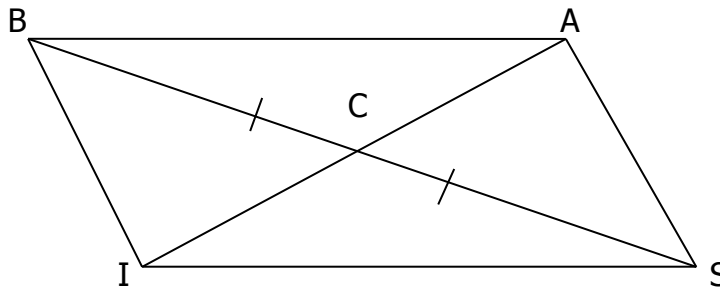
Teacher: *"From the explanation on the board, we can compare the two triangles: $\triangle ABC$ and $\triangle ADC$. We can see that the condition SSS is fulfilled."*

Teacher: *"We shall now apply the conditions for congruency of triangles explained above to solve the problems."*

He then writes the following exercise on the board:

Exercises:

Given the figure below, prove that $\triangle ABC \equiv \triangle SCI$.



Teacher: *"I give you five minutes to answer this question."*

He checks his wrist watch and when it is five minutes, he stops them from solving the problem. He also announces that only 8 minutes of the lesson remains. He asks the study participants to exchange their note books. He goes to the board and writes the following:

The study participants are looking at each other as if they do not understand the concept.

$BA = IS$ (opposite sides of a parallelogram)

$BC = CS$ (Given)

$\angle ABC = \angle CSI$ (alternate angle of a parallelogram)

Teacher: *"Hence, we have the condition SAS."*

One study participant stands up to talk.

Study participant: *"I don't understand, what is 'given'?"*

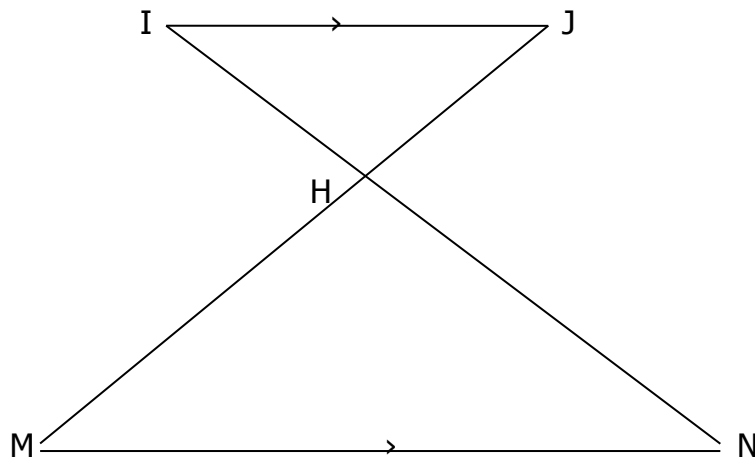
By now the lesson is about 6 minutes over the scheduled time.

Teacher: *"We shall continue from here in the next class."*

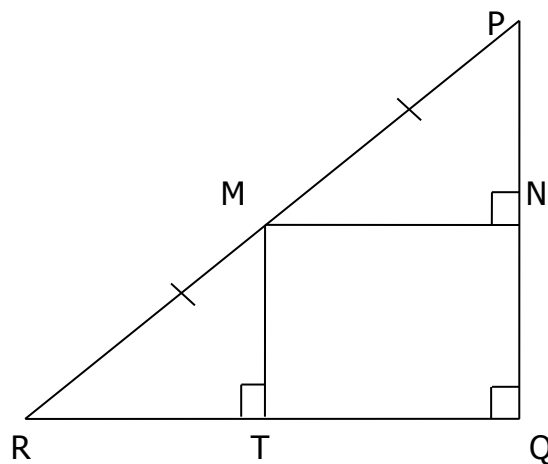
He quickly puts the following exercise on the board:

Exercise:

1. Prove that $\triangle IHJ \cong \triangle NHM$.



2. Given: $\angle Q = 90^\circ$, M is the midpoint of PR.



$MN \perp PQ$, $MT \perp RQ$. Show that:

a). Prove that $\triangle MRT \cong \triangle MNP$.

(b). Class Example: page 220

The researcher followed up on the next day's lesson. The majority of the study participants did not do the homework because they did not understand how to go about it. The teacher repeated the explanation of the previous day on the concepts of triangles as he struggled to use the homework to explain.

(ii) Results of the Data Analysis on Problem Solving Approach

(a) The number of study participants that pass each category

Table 13: The Number of Study Participants that Pass each Question Category

Category	Question Items that falls into each Category	Number of Study Participants that scored above 50%
A	(i) and (ii)	15 (54%)
B	(iii)	3 (2%)
C	(iv)	1 (1%)

Table 13 shows that 15 (54%), 3 (2%) and 1 (1%) of the study participants scored 50% and above in the question category A, B and C respectively.

(b) Solution Appraisal Data Analysis Results

The solution appraisal results are presented based on the question category.

Question Category A

Sample question is: "*Join the necessary points in the figure above to form diagonal /QR/*"

The results of the solution appraisal data analysis show that 54% of the study participants did not have a problem with locating the two corners of the shape given to be connected to form the required diagonal in question category A (see table 13 above).

Question Category B

Sample question is: "*Identify and draw $\triangle PRS$ and $\triangle QRS$ separately.*"

Only 2% of the study participants could accomplish the task given in this question category.

Question Category C

Sample question is: "If an $\hat{PQS} = \hat{RQT}$ and $\hat{QPR} = \hat{QST}$, prove that $\hat{QRP} = \hat{QTS}$."

or "Hence prove that $\triangle QPR \equiv \triangle QST$."

Only 2% of the study participants could accomplish the task given in this question category.

4.5.2 Results of the analysis of the quantitative data analysis in the Control Group

How does the Van Hiele instructional model impact on study participants' score achievement in the learning of the concepts of congruent triangles in the experimental schools?

The results emanating from the quantitative data analysis indicated that the study participants in the control group did not perform as well as their counterparts in the experimental groups in the post-test, although all the study participants performed poorly in the pre-test. The results from the data analysis are presented below:

It should be noted that the Van Hiele instructional model was not used in this group. The pedagogical approach was the traditional form of teaching. We presented the result of the traditional approach below:

(i) Results of the descriptive data analysis

Table 14: Descriptive Results in Control Group

	N	Mean	Std. Deviation	Std. Error Mean
Pre-test	28	9.46	3.305	.625
Post-test	28	26.36	8.234	1.556

In the control group, table 14 above shows that the mean score of the pre-test is 9.46%, while that of the post-test is 26.36% (rounded up to 2 decimal places). The standard deviations are 3.31% and 8.923% for the pre-test and post-test

respectively. This implies that in the pre-test, the majority of the score results fell within the (6% - 12%) category, while in the post-test, the majority of the scores fell within the (18% - 34%) category. In addition, a relatively high standard deviation of 8.23% shows that the scores are widely spread within the indicated scores category. The standard deviation error of 0.63 and 1.56 (to 2 decimal places) reflect the degree of accuracy as the mean for the data collected.

The graphical presentation of the results above enables one to see the spread of the scores both in the pre-test and post-test.

Figure 8 The Histogram of the Pre-test Results in the control group

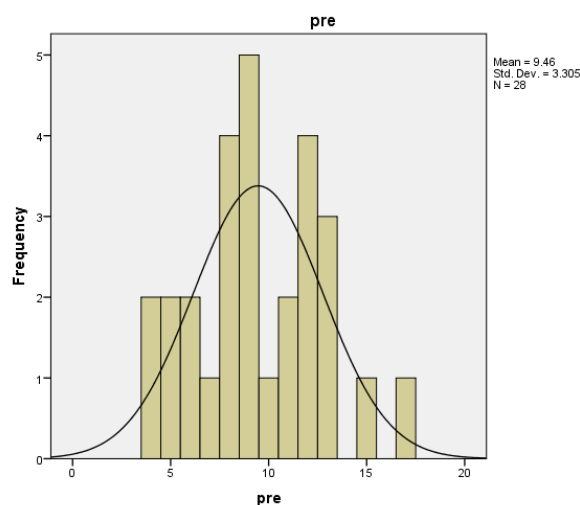


Figure 8 is a graphical summary of the results in table 8. In figure 8, it is evident that the majority of the scores results fell within the 6% and 12% category, as analyzed in table 14. The graph is also a normal distribution, which indicates that as the curve approaches zero on both the right and left sides of the graph, the limits were about 3% and 17% respectively.

Figure 9 The Histogram of the Post-test Results in Control group

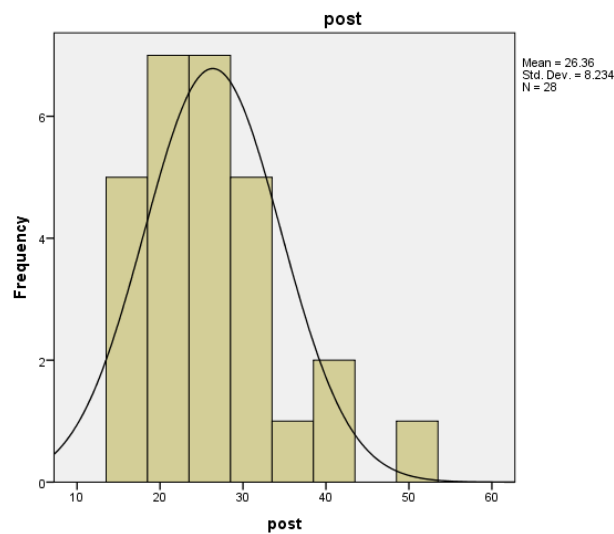


Figure 9 visually represents the results in table 14. It can be observed here that the majority of the score results spread across the (15% - 34%) category, as analyzed in table 5. In the graph as the curve approaches zero on both the right and left sides of the graph, the limits are about 10% and 56% respectively.

Even though the descriptive statistics show the improved achievement in the post-test over that of the pre-test, the results of the inferential data analysis below show the statistically significant comparison of the two means.

(ii) Results of the inferential data analysis

Table 15: One-Sample Test

	Test Value = 50					
	T	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Pre-test	-64.895	27	.000	-40.536	-41.82	-39.25
Post Test	-15.194	27	.000	-23.643	-26.84	-20.45

Table 15 shows the result of the ANOVA comparison of the means of both the pre-test and post-test, since $F < 0.005$. It is revealed that there is a statistically significant difference between the two means.

4.5.3 Comparison between the van Hiele instructional approach and the traditional approach

In this section, the results of the data analysis of the classroom observation in the control group are compared to that of the experimental group (since the classroom lesson presentation in experimental group A is similar to that of B, general results are presented for both groups A and B).

Table 16: Comparison between the classroom pedagogy in the experimental and control Groups

	Control Group (Traditional Approach)	Experimental Group (Van Hiele Teaching approach)
1	The study participants in this group got to know about the concepts of triangle congruency only when they got into the class	The teachers informed the study participants about the next topic and asked them to read, research and familiarise themselves with the topic.
2	On the first day of the lesson on congruency, the teacher went straight to the board to write and introduce the topic.	The teachers started by inquiring what the study participants know about congruency of triangles: they asked probing questions on figures and shapes properties, to find out if the study participants could link their answers to triangle congruency
3	The teacher worked examples on the chalk board to illustrate the concepts of congruency of triangles. The first lesson ended.	Based on the answers given to the probing questions, the teachers gave problems to be solved. These questions were on properties of shapes, but the properties are needed for congruency of triangles. The teacher watched the study participants' reasoning as they were attempting to solve the problems. The first lesson ended.
4	The teacher gave 5 class work exercises on what was done on the previous day and marked the study participants' work. The teacher was not satisfied with the study participants' work, he explained the concept of triangles all over again as he was solving the problems. The lesson ended.	On this day the teacher introduced the topic, clarified/confirmed/disapproved some of the study participants' ideas. Explained the concepts of congruency of triangles.
5		Wrote problems on the board and allowed the study participants to answer them in their workbook, but guided them as they attempted to solve the problems. The lesson ended.

Table 17: Comparison of how the pedagogy used affected the learning of congruence of triangles concepts in experimental and control groups

	Activities	Experimental Group	Control Group
1	Classroom Activities	The teacher asked questions that lead the study participants into thinking. Allowed study participants to research, to try things out and explore.	The teacher did most of the talking. He directed the lesson according to his prepared notes.
2	Level of study participants	Study participants participated actively throughout the time of the observed lessons.	Study participants did not contribute at all to the explanation. Where they contributed, most of the answers given to questions were wrong.
3	Quality of questions asked.	Study participants asked questions that demonstrated conceptual knowledge or such that showed their intention to seek more knowledge.	They were mostly asking for more explanations.
4	Quality of explanations given by study participants.	They gave sound conceptual explanations and showed creative thinking.	They did not participate in the explanation.
5	Performance of study participants in the formative assessments.	The majority performed well in the class work exercises.	Many of them were not doing their homework on the basis that they did not understand how to solve the problem.
6	Study participants' disposition during the classroom teaching.	They were enthusiastic and waited to participate in answering or contribute to the explanation.	Study participants in the control group always wore a confused facial expression.
7	Study participants' willingness to answer questions.	The majority were willing to answer questions.	They were not unwilling or afraid to answer questions.

CHAPTER FIVE

DISCUSSION, IMPLICATIONS, CONCLUSION AND RECOMMENDATIONS

In this chapter, the study is summarised and the findings are presented. These findings are discussed in light of the research questions, the hypothesis and the literature consulted. Conclusions and recommendations are also presented.

5. Summary of the Study

The study was conducted in selected Gauteng high schools. The Van Hiele learning model formed the conceptual framework of the study.

An intact group of the grade 10 learners from three randomly selected high schools in Gauteng formed the study participants for the study, a total of 136 learners. A mixed method approach that involved both qualitative and quantitative data collection and analysis was used to carry out the inquiry.

The study participants were divided into experimental and control groups. Intervention was used in the teaching of the concepts of the congruent triangles in the experimental groups, while the teaching followed a traditional approach in the control group.

Quantitative data were collected through pre-test and post-test design and qualitative data was collected through classroom observation. Descriptive and inferential data analyses were performed on the quantitative data, while the qualitative data was analysed through the spreadsheet. The major findings are presented below:

1. The intervention facilitated the learning of congruence of triangle concepts in the experimental groups.
2. Years of teaching experience of mathematics in group A has had a positive impact on his lesson presentation.
3. There was improvement in the geometrical thinking ability of the study participants in the experimental groups.

4. The solution approaches of the study participants in the experimental groups were improved.
5. The achievement scores of the study participants in the experimental groups were improved.
6. The Van Hiele learning level of the study participants increased.

5.1 Discussion

5.1.1 The intervention facilitated the learning of the concepts of the congruence of triangles in the experimental groups.

In all the groups, the results of the pre-test showed that the entire group did not have much knowledge in the concepts of congruency of triangles (see table 8, 11, and 14). The tables show the pre-test mean score as 15, 13, and 9 for experimental group A, B and the control group respectively. Once the teaching approaches changed, the study participants' learning trajectory of the congruence of triangles changed.

The study participants in the experimental groups participated in the teaching and learning of the concepts of congruency of triangles: they researched and explored new knowledge on the congruence of triangles. The teacher asked probing questions to help the study participants search for answers from their prior knowledge, linking concepts from their prior geometrical knowledge to the present concepts to be learnt, and they presented their cases logically (see table 11, items 2 and 3).

Perhaps this is why Halat (2008) advised that geometry teachers should plan their classroom activities in a way that can help the learners understand the nature and concepts of geometry (see subsection 1.3.2 of this work that shows how Van Hiele instructionals are arranged). The teachers in the experimental groups followed the Van Hiele instruction approach, which details the steps to take when teaching geometry. The steps are listed in subsection 1.2.2 of this work.

In addition, Halat (2008) also argued that geometrical knowledge transfer should be systematic so that concepts are ordered. The results of the data analysis suggested that the intervention might have influenced active classroom participation in the

experimental classes. That is, the intervention might have facilitated the learning of congruence of triangles. In the experimental groups A and B, the effect sizes were 2.3 and 1.97 respectively, which implies that the probability that the intervention was responsible for these results is very high.

This result is also in line with the results found by *Erdoğan* et al. (2009), Malasia, Abdul and Zakaria (2013) and Mohd Salleh et al. (2013). The study participants in the control group were not active and only listened to the teacher. Item 6 in table 9 shows that study participants in the control group were confused in the class and item 5 also shows that they could not do their homework. These findings suggested that the traditional approach used in the teaching of congruence of triangles in the control group might have complicated learning. The researcher would have loved to conduct interviews to confirm why the study participants could not do their homework, however, this was not possible due to a lack of time and it not being part of the research design. It is assumed that the majority of the study participants involved in the control group probably lacked adequate conceptual ability to do the homework.

5.1.2 Years of teaching experience has an impact on how mathematics teachers deliver their lesson

One of the findings in this study is that some of the study participants in the experimental group B sometimes looked confused by the expression on their faces and the fact that they were silent. This occurred when tasks given by the teacher required a high level of cognitive thinking. Slavin (1996) warns that a lack of learning occurs when students are confused. The researcher tried to link this to other parts of the results that showed that the teacher in experimental group B was not patient in delivering his lessons and table 6 shows that this teacher had only 3 years of teaching experience, compared to the teacher in the experimental group A, who had 23 years of mathematics teaching experience. How the teachers delivered the intervention in their schools reflected in both the effect of size and achievement results. The experimental group A had 2.3 effect size with a mean achievement score of 61% while experimental group B had the effect size of 1.97 with a mean achievement score of 41%.

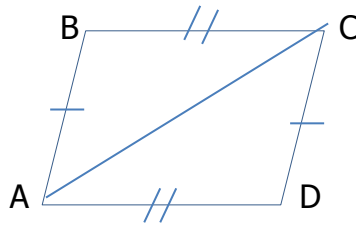
It is possible that, since both the teachers used the same intervention, the larger effect size in experimental group A might have occurred because of the way the teacher in experimental group A presented his lesson, which might have been informed by his age and teaching experience.

5.1.3 There was improvement in the geometric thinking ability of the study participants in the experimental groups

The findings in the experimental groups indicated that the study participants presented logical arguments when answering questions, and when solving problems in the classroom. They responded very well to the probing questions the teacher was putting forward. This was especially clear from how the study participants argued the class work given in the experiment group A.

One of the study participants presented a solution by extending the sides of the parallelogram to find the solution, while another study participant used the equal given sides and the properties of the diagonal of the parallelogram to arrive at the answer. The researcher observed that there was active mental thinking going on among the study participants once the lesson commenced in the experimental groups. The findings presented in table 16 (items 2 to 4) show that the study participants gave responses that implied the study participants engaged conceptual geometric thinking while the intervention was going on. In addition, tables 7 and 10 show that the majority of the study participants in the experimental groups were able to solve the problems in question in category B and C, which required more geometrical conceptual thinking. These findings conform to literature such as *Erdoğan et al. (2009)*, who also used the Van Hiele instructional approach to improve learners' creative thinking. *Abu and Zaid (2013)* also agrees with this school of thought.

On the other hand, the study participants in the control group were not able to solve similar problems given to them. The question was to show that $\triangle ABD$ and $\triangle ADC$ is congruent given the parallelogram below:



The teacher went as far as splitting the triangles but the study participants could not connect the concepts stated in the congruency conditions to solve this problem until the teacher gave the answer. These results are again in line with the literatures cited in chapter 2, which are *Erdoğan et al.*, (2009), Meng (2009) and Alex and Mammen (2012).

Contrary to the findings in the experimental groups, findings in the control group indicated that the study participants were not able to apply logical thinking. In addition, table 17 (item 6) informs that the study participants appeared confused during the lesson observed. This is a sign that they were not learning. In this situation, any other side activities (non-academic) could have distracted them. This is supported by Howie's (2001) findings that South African children are easily distracted when confronted by questions in geometry. Slavin (1996) warns that conceptual retention happens when a learner pays attention in the class. That was probably the reason why the study participants in this group could not do their homework because they did not understand how to go about solving the problems.

5.1.4 Improvement in the solution approaches of the experimental groups

The data analysis shows that in group A, 41 (86%), 33 (69%) and 63 (59%) of the study participants scored 50% and above in the question category A, B and C (see table 7), while it was found that in group B, 53 (84%), 35 (59%) and 31 (52%) of the study participants scored 50% and above in the question categories A, B and C, (see table 10). These findings show that the majority of the study participants were able to apply appropriate concepts to solve geometrical problems that require abstract thinking. In addition, questions (ii) and (iii) of the post-test (see appendix 8) are of high cognitive demand.

Though some of the study participants could not answer 1(ii) completely but made good attempts by being able to see that if $Q\hat{R}P = Q\hat{T}S$, they could show that $\Delta QPR = \Delta QST$. See figure 3, which is a sample script of the approach the study participants applied in solving the post-test problems.

It is evident from the sample script that the study participants were able to identify and draw the required triangles from the figure given, but the study participants struggled to prove that $Q\hat{R}P = Q\hat{T}S$. This question required deep conceptual thinking. The use of the sum of angles in a triangle concept is vital in solving this problem (see the memo, appendix 9). This was how far the study participants could think to solve the problem.

However, it was also found that 15 (54%), 3 (2%) and 1 (1%) of the study participants in the control group scored 50% and above in the question categories A, B and C (see table 13). It was noted that the majority of study participants in the control group could not apply the critical thinking that the questions in category B and C required.

5.1.5 Improve the achievement score of the study participants in the learning of congruent triangles

As noted in section 3.1, the intervention was only limited to experimental groups A and B, while in the control group, the teacher taught the class using a traditional chalk-and-talk, teacher-centred teaching approach. Descriptive data analysis revealed that, in the experimental group A and group B, the mean achievement scores were 61% and 41% (see table 8 and 11). As reported in sub-section 4.3.1 and 4.3.2, the implication of these mean scores with the standard deviation is that the majority of the study participants scored between 44% to 78% in experimental group A, and 34% to 48% in experimental group B in the post-test. In the control group the mean achievement score in the post-test was 26 and the majority of the study participants in this group scored between 18% and 34% in the post-test (see table 14). It is evident that the majority of the study participants in the intervention groups (i.e A and B) achieved better results than the study participants in the control group, particularly in group A.

South African high school learners traditionally perform very poorly when it comes to learning geometry. As noted in Chapter 2 of this study, Howie (2001) informed that South African learners have difficulty in dealing with geometry questions in the TIMSS-R studies. This trend was confirmed by the learners in the control group, where the majority of them fell within the 18%-34% achievement category in the pre-test. In the experimental groups on the other hand, the majority of the study participants had higher achievement scores (see figures 5, 7, and 9). These results suggest that the intervention might have been responsible for the improvement in the achievement scores in the experimental groups.

The results of the inferential statistics also confirm the statistically significant improvement when comparing the achievement scores in both experimental groups A and group B with $F < 0.005$. It is noted that the slight increase between the pre-test and the post-test in the control group is also significant but it was explained above that the majority of the study participants here scored between 15% and 34%. This is illustrated visually in figure 9, which shows the spread of the performance of the study participants in the control group in the post-test.

However, for each of these results (in group A, group B and the control group), the data analysis of the qualitative data complimented them all. In experimental group A, the teacher was very patient with study participants, for example he allowed them to think rather than hurry them up. This could have been due to his age and years of experience in teaching (23 years). It is therefore noted that being patient in handling learners and going about the intervention in a subtle manner might have contributed to the best achievement being recorded in experimental group A.

The experimental group B teacher though, taught according to the Van Hiele instructional approach but he was a little hard on the study participants. This, together with the large number of the study participants in the class (60 study participants) and a relatively small classroom, might have been the reason for the low achievement in this group. Some of the study participants were making a noise, others were not listening and some were even sleeping. The researcher was of the

opinion that those that were paying attention were the ones with the best achievements (some got above 50%).

These results are in line with the existing literature. Slavin (1996) refers to the work of Bandura (1977) (cited in Chapter 2), which infers that learning involves four phases: attention, retention, reproduction and motivation.

Kotze (2007), Atebe (2008) and Connolly 2010, as cited in section 2, demonstrated the potency of the Van Hiele instructional model to improve students' achievement in geometry. They might have proved De Villiers (1997) and Roux (2003) right by implying that South African schools should adopt the Van Hiele instructional approach in the classroom teaching of geometry.

5.1.6 The Van Hiele level of the study participants in the experimental group increased.

Figure 4 shows the spread of the performance of the study participants in the experimental group A in the pre-test, while table 6 shows the mean of these scores as 15%. Likewise, figure 6 shows the spread of the performance of the study participants in experimental group B in the pre-test, while table 11 shows the mean of these scores as 13.6%. It was evident in the pre-test scripts that the study participants were only able to attempt questions (i) and/or (ii), which were in the question category A and corresponds to the Van Hiele level 1 (see table 5).

Comparatively, figure 5 shows the spread of the performance of the study participants in the experimental group A in the post-test, while table 8 shows the mean of these scores as 61%. Likewise, figure 7 shows the spread of the performance of the study participants in experimental group B in the pre-test, while table 11 shows the mean of these scores as 41%. In addition, table 7 shows that 56% of the study participants in experimental group A got 50% and above in question category C, which is considered to be Van Hiele level 3. Similarly, table 10 shows that 52% of the study participants in the experimental group B got 50% and above in the question category C.

This implies that the Van Hiele learning level of the study participants moved from level 1 to level 3. However, the study participants in the control group could hardly solve the problem in category B, which is a combination of Van Hiele learning level 1 and 2. The results are in line with the findings of Alex and Mammen (2012) and Malasia, Abdul and Zakaria (2013) in which the van Hiele instructional model was used to move study participants in the experimental groups from one level of van Hiele geometric thinking to a higher level.

5.1.7 The effect of teaching the congruency of triangles with the Van Hiele instructional model on the study participants' understanding of the concepts of the congruence of triangles.

The results of the data analysis show that, based on classroom observations, the study participants from the experimental groups A and B understood the concepts of congruence of triangles taught, while the study participants in the control group struggled to understand the concept.

The result also show that in the experimental groups, the study participants were asking constructive questions, explaining the concept to the extent that they understood it, and were really involved in the teaching. After the 'direct orientation' and 'making clear' stages of the teaching, the majority of the study participants were able to solve problems on their own, although with some challenges. It was clear in the experimental group that the majority attempted their homework intelligently and they had good marks in their homework, which was probably the reason why they performed well in the post-test. This result conforms to the findings of Howie (2001) and Slavin (1996), which informs that a learner demonstrates deep conceptual understanding of the topic taught when the learner can reproduce what was learnt in class and is able to solve problems.

On the other hand, the study participants in the control group were unresponsive as the lessons observed were teacher dominated and the study participants were like an empty barrel into which the teacher continued to pour his knowledge. The questions raised by the study participants showed that they were struggling to understand the concepts taught. For example, when the teacher presented the

conditions for the congruency of triangles, one of the study participants asked if they would be measuring the sides of the triangles to show that they are equal.

The teacher simply answered that they would be given. It should be observed that the study participants in this group were struggling to understand but they were not taken through the route in which they could take a lead in the teaching and learning. This could be the reason why Kalu (2010) remarked that traditional mathematics classroom pedagogy produces learners whose performance in mathematics is poor in mathematics concept knowledge and that they are inadequately equipped with critical problem solving skills.

5.2 Conclusion

The results that emerged from this study suggest that if the Van Hiele instructional model is affected in the teaching of congruency of triangles in the grade 10 mathematics classroom, it may facilitate the process of learning the concepts taught and improve the achievement scores of the learners. This may not be limited to teaching only congruency of triangles in geometry, which is why De Villiers (1997, 2006) consistently remark that the South African curriculum should adopt the Van Hiele instructional approach in the classroom teaching of geometry. The researcher trusts that the education stakeholders will consider the outcomes of this study to improve the state of the teaching and learning of geometry in the South African schools.

5.3 Recommendations

The researcher recommends that the Van Hiele learning and instructional model be adopted and applied in the teaching of other areas of mathematics as well. This might be the solution to the continual poor performance of learner's in mathematics.

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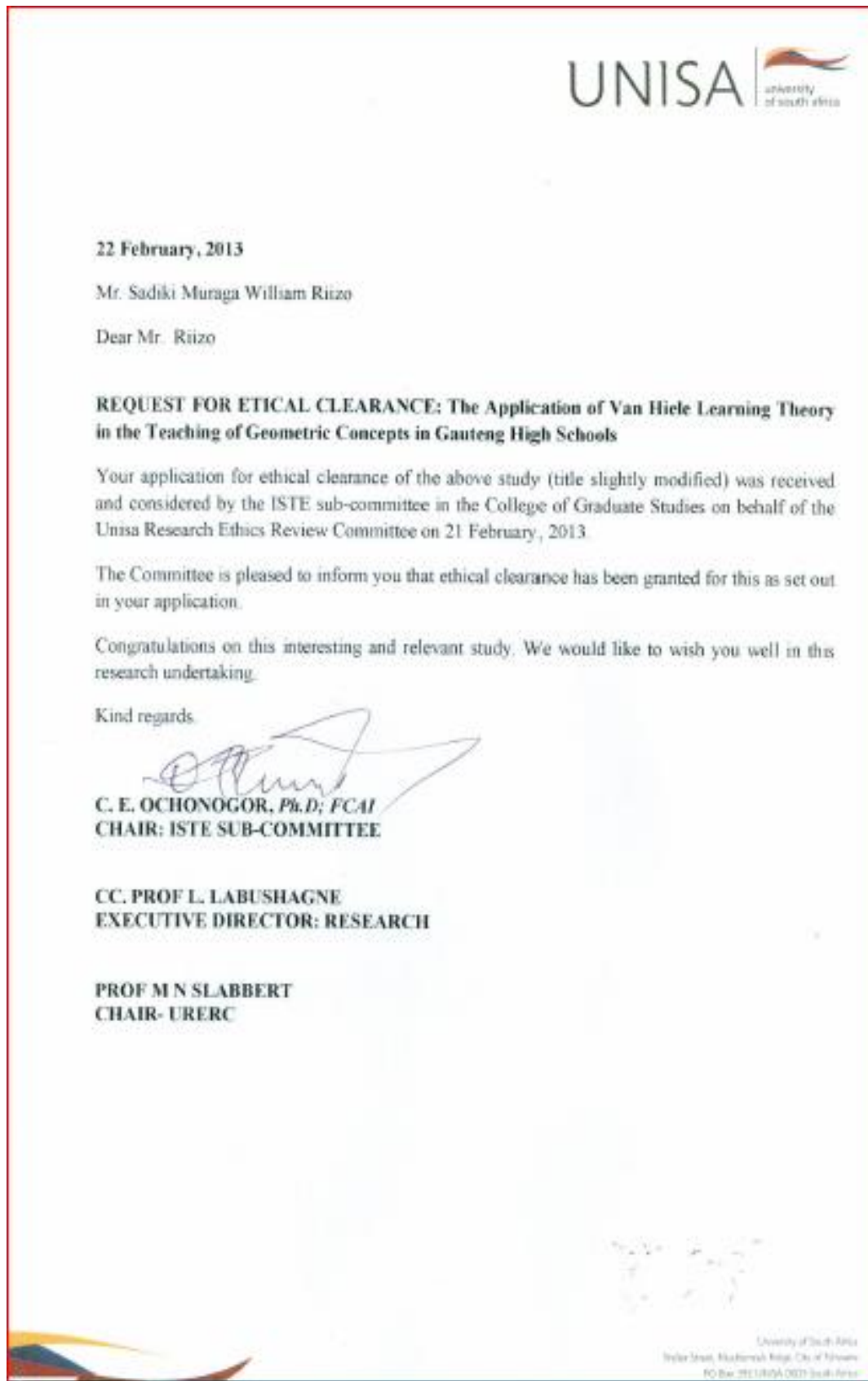
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APPENDIX 1



APPENDIX 2

	GAUTENG PROVINCE Department: Education REPUBLIC OF SOUTH AFRICA
Reference : Policy and Planning: Partnerships Enquiries : Sello George Ngwenya Telephone : 012 401 6322 Fax : 012 401 6323 E-mail : Sello.Ngwenya@gauteng.gov.za	
11 March 2013	

Sadiki Muraga William Ruzo
3700 Vanadium Street
Clayville Ext 33
Olifantsfontein,
Midrand, 1666
Mobile : 082 219 7643; 084 350 0407; Fax: 086 580 4072/67; 086 762 2277
e-mail: ;

Cc: The Principal and SGB

Dear Sir/ Madam

PERMISSION TO CONDUCT RESEARCH: SADIKI M.W

Your research application has been approved by Head Office. The full title of your Research: "The application of van Hiele learning theory in the teaching of geometry concepts in Gauteng High schools: Connecting the theory to Practical". You are expected to adhere strictly to the conditions given by Head Office. You are also advised to communicate with the school principal/s and/or SGB/s of the targeted schools regarding your research and time schedule.

Our commitment of support may be rescinded if any form of irregularity/ no compliance to the terms in this letter or any other departmental directive/ if any risk to any person/s or property or our reputation is realised, observed or reported.


Terms and conditions

1. The safety of all the learners and staff at the school must be ensured at all times.
2. All safety precautions must be taken by the researcher and the school. The Department of Education may not be held accountable for any injury or damage to property or any person/s resulting from this process. The school/s must ensure that sound measures are put in place to protect the wellness of the researcher and his/ her property.

NB Kindly submit your report including findings and recommendations to the District at least two weeks after conclusion of the research. You may be requested to participate in the Department of Education's mini-research conference to discuss your findings and recommendations with departmental officials and other researchers.

The District wishes you well.

Yours sincerely


Mrs. H.E. Kekana
Director: Tshwane South District
Date : 14/03/2013

Making education a societal priority
Office of the District Director: Tshwane South District
(Mamelodi/Eensterust/Pretoria/Pretoria East/Pretoria South/Atteridgeville/Laudium)
255 Pretorius Street, Pretoria 0001
Private Bag X 166, Pretoria, 0132 4016300 Fax: (012) 4016318
Website: www.education.gpg.gov.za

Submitted to Mr. Muraga Sadiki on

APPENDIX 3

LEARNER'S INFORMED CONSENT

Information for Research Participant

My name is Mr Sadiki Muraga, I am a UNISA postgraduate student (student number 36636479), I am conducting an academic research on the topic: **The Effect of Using Van Hiele Instructional Module in the Teaching of Congruent Traingles in Grade-10 in Gauteng High Schools.** As the research topic implies, the purpose of the study is to determine the extent to which Van Hiele Instructional Model may improve the learning of congruent triangles Grade 10 mathematics classes.

It is in the light of this that your consent to participate in the research work has been sought. Please, note that any information supplied shall remain strictly confidential and anonymous, and shall be used for the purpose of this investigation only. If you are willing to participate in the research, please sign this informed consent form. Thank you for your interest and cooperation.

Researcher's **Name:** _____

Signature: _____ **Date:** _____

APPENDIX 4

Participant's Declaration

I (optional) hereby confirm that I have been well-informed by the researcher about the nature, conduct, benefits and risks of the study. I have also read and understood the above information. I am aware that the outcome of the study shall be anonymously processed into a research report. I understand that my participation is voluntary and that I can, at any level of the study, without prejudice, withdraw my consent and participation in the study. I had sufficient opportunity to ask questions and therefore, of my own volition, declare my intention to participate in the study.

Research **P**articipant's **S**ignature: _____ Date: _____

APPENDIX 5

PARENTAL INFORMED CONSENT FORM

(Applicable where the participant is younger than 18 years)

I hereby confirm that I have been well informed by the researcher about the nature, conduct, benefits and risks of the study. I have also read and understood the information about the study as contained in the Learner's Informed Consent. I am aware that the outcome of the study, and my child's personal details, will be anonymously processed into a research report. I understand that his/her participation is voluntary and that he/she may, at any level of the study, without prejudice, withdraw his/her consent and participation in the study. He/she has had sufficient opportunity to ask questions and I, of my own volition, declare that my child can participate in the study.

Research **P**articipant's **N**ame: _____

Name of Research **P**articipant's **P**arent/**G**uardian: _____

Signature of Research **P**articipant's **P**arent/**G**uardian: _____

Date: _____

Researcher's name: _____

Researcher's **S**ignature: _____ Date: _____

APPENDIX 6

CLASSROOM OBSERVATION CHECKLIST

(Designed According to Van Hiele Instructional Model)

Observer: _____

School: _____

Educator: _____

Grade: _____ Number of Learners in the Class: _____

Topic Taught: _____

Date of Observation: _____ Time: _____

Scale: Yes, No, or N/A

Please note: N/A means Not Applicable

A. Classroom Organisation	Yes	No	N/A
---------------------------	-----	----	-----

1. The classroom is very spacious
2. Learners are comfortably seated.
3. The board is a white marker board
4. Participants were seated in groups

B. Lesson Presentation

1. Teacher begins class at the appropriate time.
2. Materials presented are appropriate to the level of learners
3. Materials presented are related to the objectives of the learning area.
4. Teaching Approach

C. Lesson Presentation Procedure

1. Teacher already informed the study participants about the topic to be taught prior to the lesson to allow learners to research about it.
2. Teacher post questions to determine the geometric thinking level of the study participants

3. Teacher gives a class work to be discussed in each group.
4. Teacher introduces the day's topic and give brief explanation on the topic for clarification.
5. Teacher give a more cognitive demanding class work to be solved by study participants.
6. Teacher allows the study participants to demonstrate their geometric thinking abilities and also goes round the class to see what each group were doing, correct them, confirm their solution approaches or give more explanation.
7. Teacher allows the study participants to explain how each group went about Solving the problem, while the teacher and other leaner's ask questions.
8. Teacher gives exercises and/or homework.

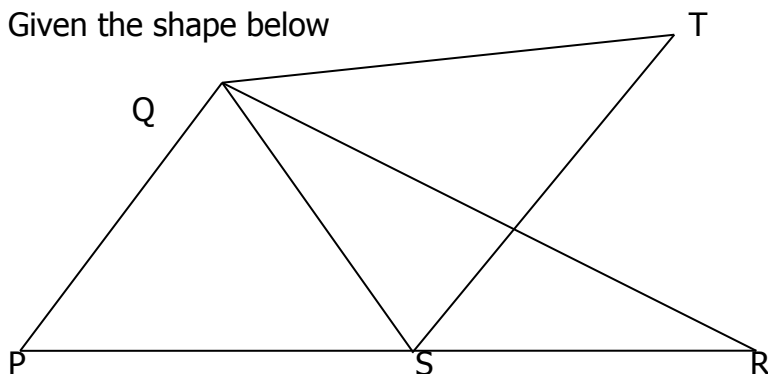
APPENDIX 7

Pre-test

Total Marks: 60

Work out the following:

1. Given the shape below



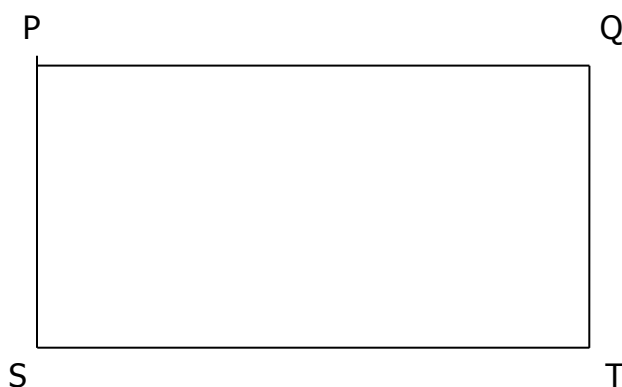
- (i). Identify and draw $\triangle QPR$ and $\triangle QST$ separately. (4)

- (ii). If $\angle PQS = \angle RQT$ and $\angle QPR = \angle QST$, prove that $\angle QRP = \angle QTS$ (10)

- (iii). Hence prove that $\triangle QPR \equiv \triangle QST$. (4)

[18]

2. PQRS is a rectangle.



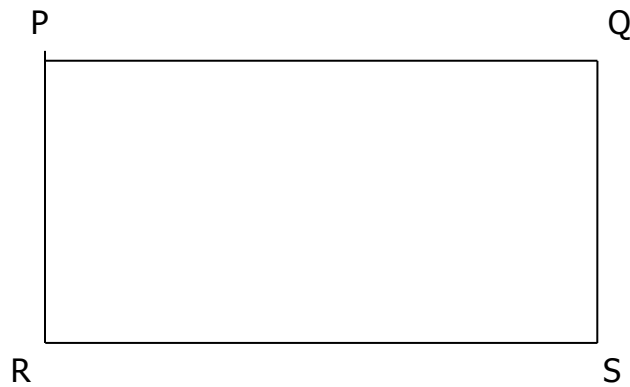
- (i). Join the necessary point in the figure above to form the diagonal [QR]. (2)

- (ii). Identify and draw $\triangle PRS$ and $\triangle QRS$ separately. (4)

- (iii). Use the properties of rectangles (10)

[16]

3. PQRS is a rectangle.



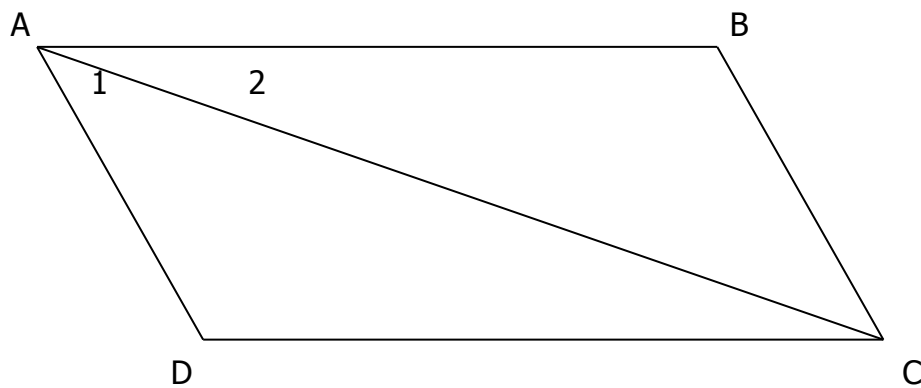
(i). Join the diagonal lines $|PR|$ and $|QS|$. (2)

(ii). Identify and Sketch $\triangle PRS$ and $\triangle QRS$ separately. (4)

(iii). Use the properties of rectangles (10)

[16]

4. ABCD is a parallelogram with $AB = DC$ and $AD = BC$



(i). Identify and draw $\triangle ABC$ and $\triangle ADC$ separately. (2)

(ii). Use the properties of parallelogram to show that $|AB| = |DC|$. (6)

(iii). And hence prove that $\triangle ABC \equiv \triangle ACD$ (8)

[16]

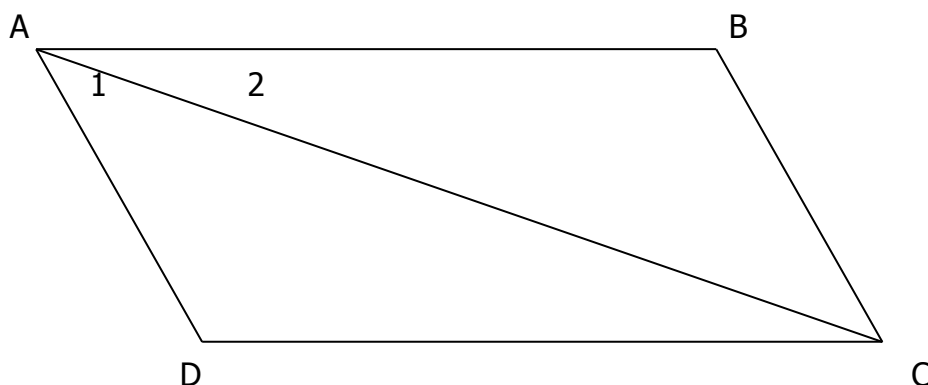
APPENDIX 8

Post-test

Total Marks: 60

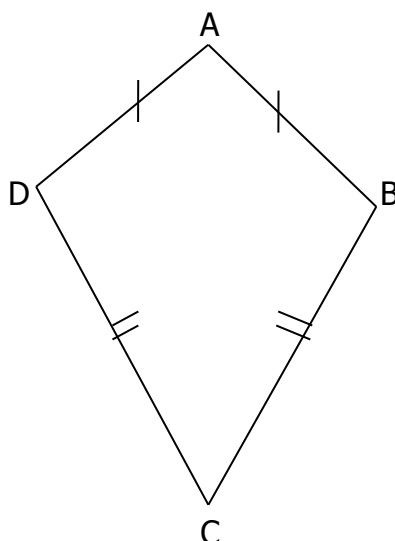
Work out the following:

5. ABCD is a parallelogram with $AB = DC$ and $AD = BC$



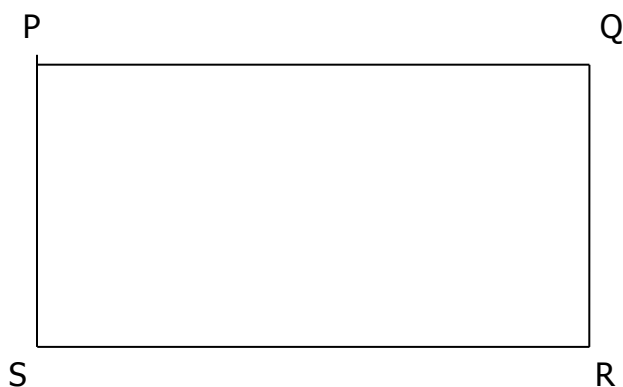
- (i). Identify and draw $\triangle ABC$ and $\triangle ADC$ separately. (2)
- (ii). Use the properties of parallelogram to show that $|AB| = |DC|$. (6)
- (iii). And hence prove that $\triangle ABC \equiv \triangle ACD$ [8]

6. ABCD is a kite with $AD = AB$ and $CD = CB$



- (i). Join the diagonal line AC. (2)
- (ii). Draw $\triangle ABC$ and $\triangle ADC$ separately (2)
- (iii). Prove that $\triangle ABC \equiv \triangle ADC$. (6)

7. PQRS is a rectangle.



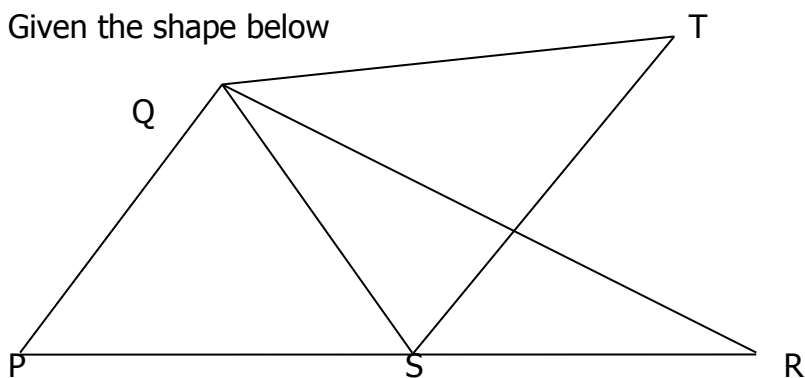
(i). Join the diagonal lines $|PR|$ and $|QS|$. (2)

(ii). Identify and draw $\triangle PRS$ and $\triangle QRS$ separately. (4)

(iii). Use the properties of rectangles (10)

[16]

8. Given the shape below



(i). Identify and draw $\triangle QPR$ and $\triangle QST$. (4)

(ii). If $\hat{PQS} = \hat{RTS}$ and $QPS = QST$, prove that $QRP = QTS$. (10)

(iii). Hence prove that $\triangle QPR \equiv \triangle QST$. (4)

[18]

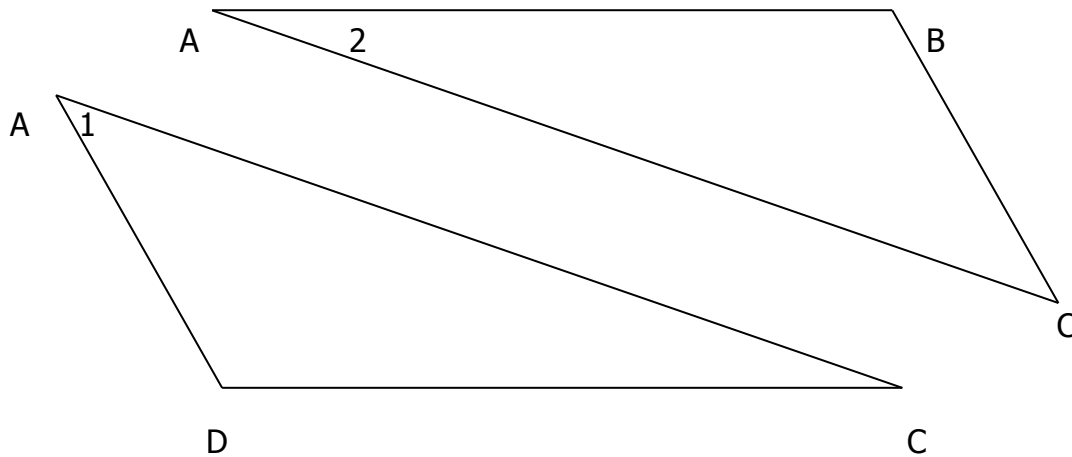
APPENDIX 9

Post-test Memorandum

Marks: 60

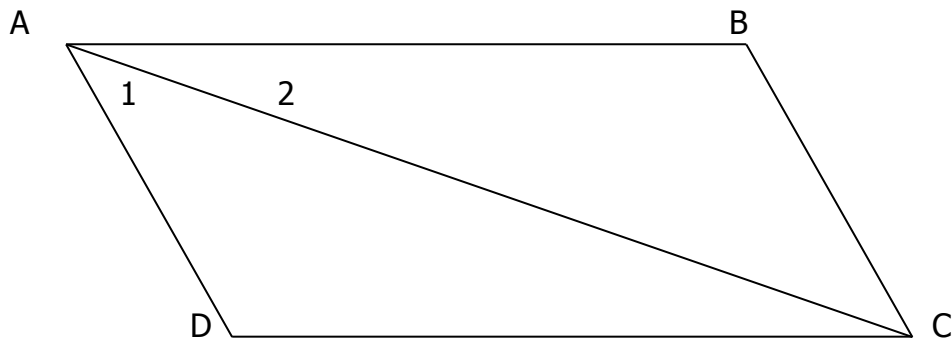
Question 1.

(i).



(ii). Yes, $|AB| = |DC|$ Opposite side of a Parallelogram

(iii).



$|AD| = |BC|$ Opposite side of parallelogram

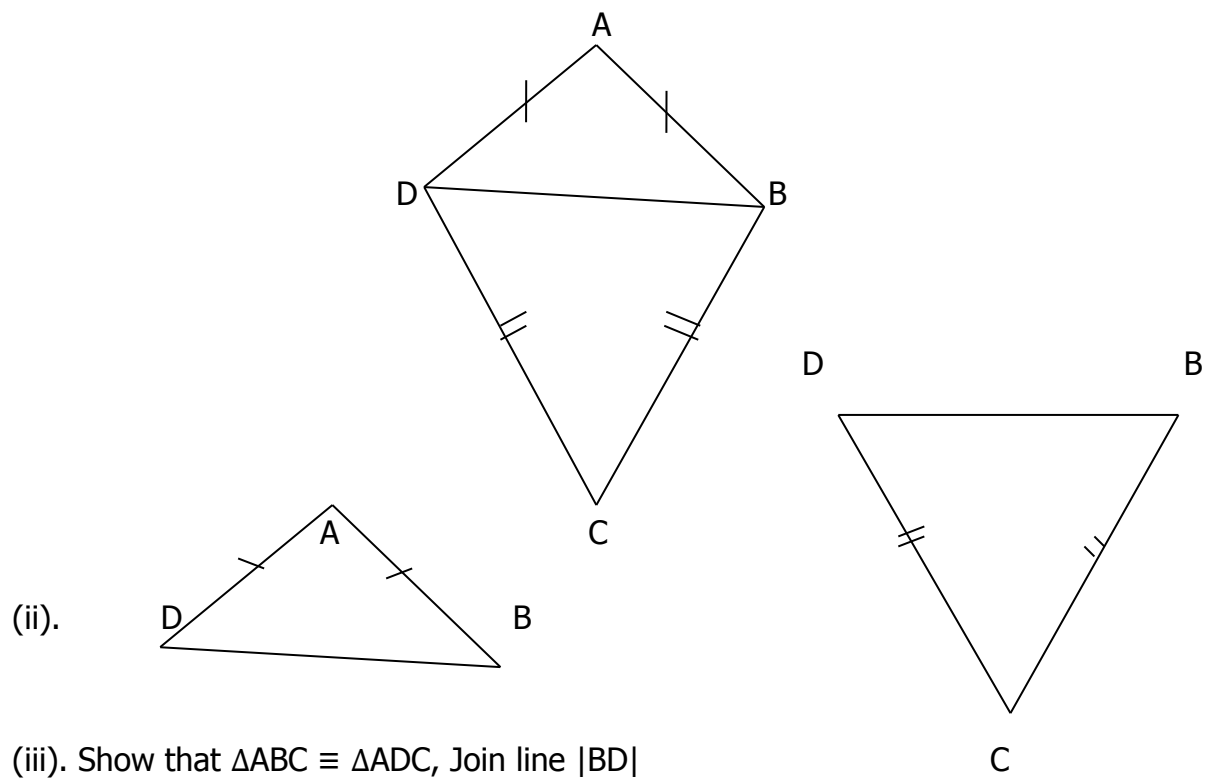
$|AB| = |CD|$ Opposite side of parallelogram

$\hat{ABC} = \hat{ADC}$ Opposite angles of parallelogram

Hence, $\triangle ABC \equiv \triangle ADC$ (S, A, S)

Question 2.

(i). ABCD is a kite with $AD = AB$ and $CD = CB$



$$\hat{BDC} = \hat{CBD}.$$

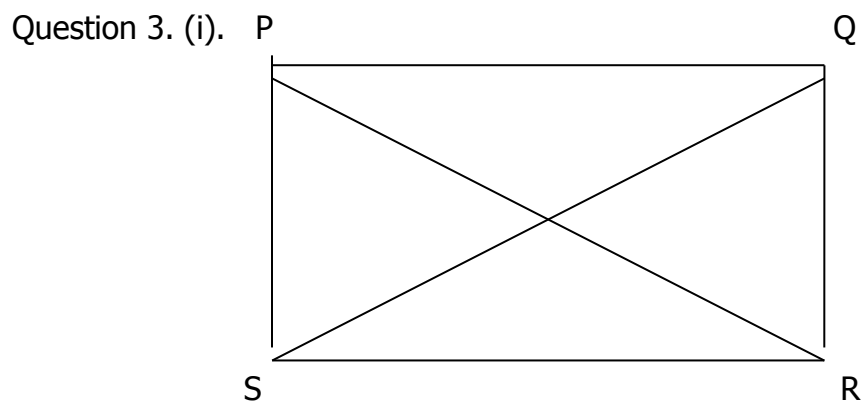
$$\text{Similarly, } \hat{ADB} = \hat{ABD}$$

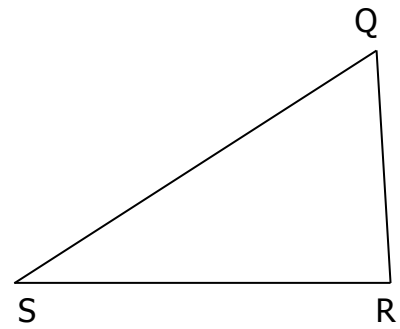
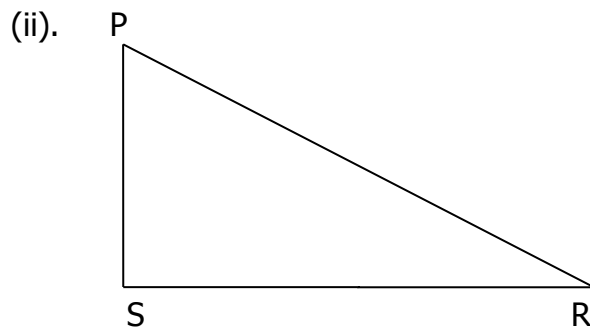
$$\text{Therefore } \hat{ADC} = \hat{ABC}$$

$$\text{Since } |AB| = |AD|$$

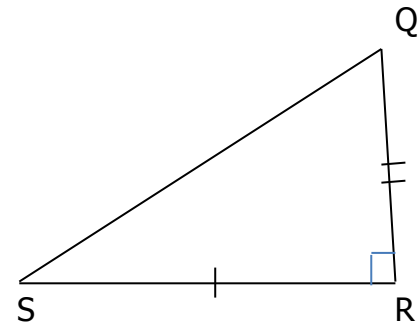
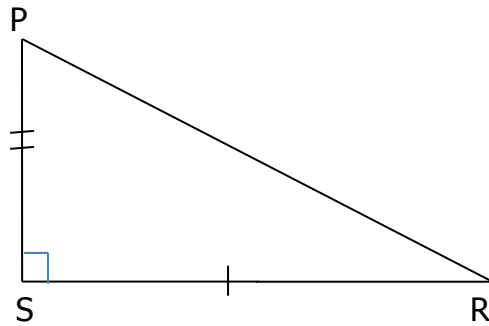
$$|BC| = |DC|$$

$$\text{Therefore } \triangle ABC \cong \triangle ADC \quad (\text{A, S, A})$$





(iii).



$|PS| = |QR|$ Opposite side of a rectangle

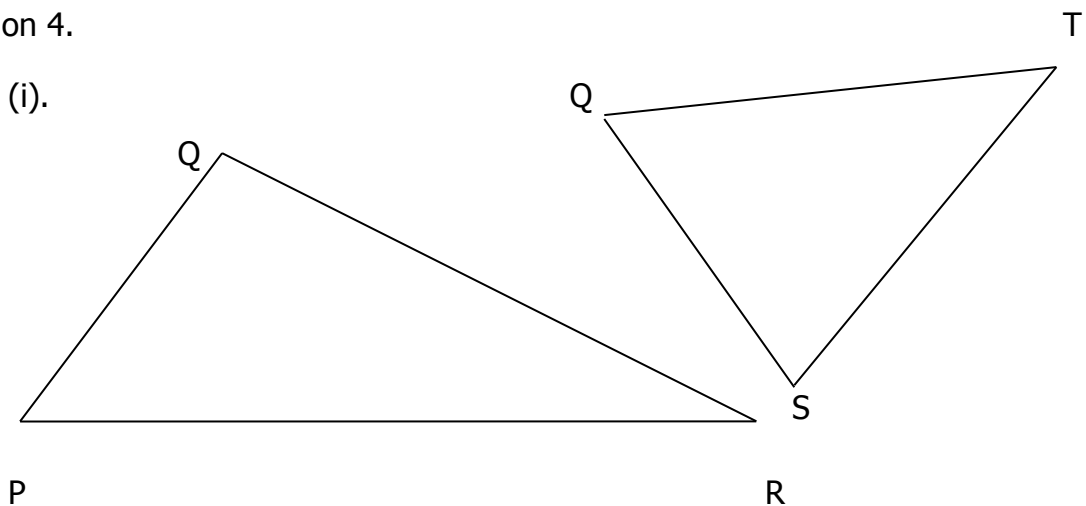
$|RS| = |SR|$ Common side

$|PR| = |SR|$ Opposite side of a rectangle

Therefore $\hat{PSR} = \hat{QRS}$ (S, A, S)

Question 4.

1. (i).



(ii). Consider $\triangle QPR$

$$\hat{QPR} + \hat{PQR} + \hat{PRQ} = 180$$

$$\text{But } \hat{PQR} = \hat{PQS} + \hat{SQR}.$$

Therefore $\hat{QPR} + (\hat{PQS} + \hat{SQR}) + \hat{PRQ} = 180$

Therefore $\hat{PRQ} = 180 - [(\hat{PQS} + \hat{SQR}) + \hat{QPR}]$

$$\hat{PRQ} = 180 - [\hat{PQS} + \hat{SQR} + \hat{QPR}]$$

(1)

Consider ΔSQT

$$\hat{QST} + \hat{SQT} + \hat{STQ} = 180$$

$$\text{But } \hat{SQT} = \hat{SQR} + \hat{RQT}$$

$$\text{Therefore } \hat{QST} + (\hat{SQR} + \hat{RQT}) + \hat{STQ} = 180$$

$$\text{Therefore } \hat{STQ} = 180 - [(\hat{QST} + \hat{SQR}) + \hat{RQT}]$$

$$\hat{STQ} = 180 - [\hat{QST} + \hat{SQR} + \hat{RQT}]$$

(2)

Consider equation (1)

$$\hat{PRQ} = 180 - [\hat{PQS} + \hat{SQR} + \hat{QPR}]$$

$$\text{But } \hat{PRQ} = 180 - [\hat{PQS} + \hat{SQR} + \hat{QST}]$$

$$\text{Similarly } \hat{PQS} = \hat{RQT} \quad \text{given}$$

$$\text{Therefore } \hat{PRQ} = 180 - [\hat{RQT} + \hat{SQR} + \hat{QST}]$$

$$= \hat{STQ}$$

$$\text{Therefore } \hat{PRQ} = \hat{STQ}$$

$$\text{(iii). } \hat{QPR} = \hat{QST}$$

$$\hat{PRQ} = \hat{STQ}$$

$$|BC| = |DC|$$

Therefore, $\triangle QPR \equiv \triangle QST$.

APPENDIX 10

Effect Size (ES) calculations

$$ES = \frac{\bar{x}_{\text{exp}} - \bar{x}_{\text{control}}}{SD_{\text{pooled}}}$$

Where: ES = Effect size

\bar{x}_{exp} = mean of the scores in the experimental group

\bar{x}_{control} = mean of the scores in the experimental group

SD_{pooled} = standard deviation using the experimental and control group data

SD_{pooled} is calculated as follows:

$$SD_{\text{pooled}} = \sqrt{\frac{(n_{\text{exp}} - 1)(SD_{\text{exp}})^2 + (n_{\text{control}} - 1)(SD_{\text{control}})^2}{n_{\text{exp}} + n_{\text{control}} - 2}}$$

Where: n_{exp} = sample size for the experimental group

n_{control} = sample size for the control group

SD_{exp} = standard deviation for the experimental group

SD_{control} = standard deviation for the control group

Effect size for Experiment group A

Where $n_{\text{exp}} = 28$

$n_{\text{control}} = 18$

$SD_{\text{exp}} = 28$

$SD_{\text{control}} = 8.2$

$$SD_{\text{pooled}} = \sqrt{\frac{(48-1)(18)^2 + (28-1)(8.2)^2}{48+28-2}}$$

$$= \sqrt{\frac{17043.48}{74}} = \sqrt{230.32} = 15.18$$

$$\text{then } ES = \frac{61.25 - 26.36}{15.18} = 2.3$$

Effect size for Experiment group B

$$\text{Where } n_{\text{exp}} = 60$$

$$n_{\text{control}} = 7.2$$

$$SD_{\text{exp}} = 28$$

$$SD_{\text{control}} = 8.2$$

$$SD_{\text{pooled}} = \sqrt{\frac{(60-1)(7.2)^2 + (28-1)(8.2)^2}{60+28-2}}$$

$$= \sqrt{\frac{4874.04}{86}} = \sqrt{56.67} = 7.53$$

Then

$$ES = \frac{41.23 - 26.36}{7.53} = 1.97$$

APPENDIX 11

DATE	TOPIC	CONTENT	F	ASSESSMENT	DATE	%
					Completed	Completed
TERM 1			2 TASKS FOR TERM 1			
09/1 – 11/1 (3 days)	Algebraic expressions	<ul style="list-style-type: none"> Understand that real numbers can be rational or irrational. Simplify expressions using the laws of exponents for rational exponents. Establish between which two integers a given simple surd lies. 				3.1%
14/1 – 18/1	Algebraic expressions	<ul style="list-style-type: none"> Round real numbers to an appropriate degree of accuracy. Multiplication of a binomial by a trinomial. Factorisation: Trinomials. 				6.3%
21/1 – 25/1	Algebraic expressions	<ul style="list-style-type: none"> Factorisation: Grouping in terms. Sum and difference of two cubes. Algebraic fractions: Denominator with monomial, binomial and trinomial terms. <p>(limited to sum & difference of cubes)</p>		<div style="display: flex; align-items: center; justify-content: center;"> <div style="font-size: 3em; margin-right: 10px;">}</div> <div style="text-align: center;"> INVESTIGATION OR PROJECT </div> </div>		9.4%
28/1 – 01/2	Algebraic expressions	<ul style="list-style-type: none"> Factorisation: Grouping in terms. Sum and difference of two cubes. Algebraic fractions: Denominator with monomial, binomial and trinomial terms. <p>(limited to sum & difference of cubes)</p>	F			12.5%
04/2 – 08/2	Exponents	<ul style="list-style-type: none"> Revise laws of exponents where $x, y > 0$ and $m, n \in \mathbb{Z}$ Use the laws of exponents to simplify expressions and solve equations, accepting that the rules also hold for $m, n \in \mathbb{Q}$ 				15.6%
11/2 – 15/2	Exponents	<ul style="list-style-type: none"> Use the laws of exponents to simplify expressions and solve equations, accepting that the rules also hold for $m, n \in \mathbb{Q}$ Exponential equations 				18.8%
18/2 - 22/2	Number patterns	<ul style="list-style-type: none"> Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore linear. <p>WITHOUT USING A FORMULA</p>				21.9%

25/2 - 01/3	Equations and Inequalities	<ul style="list-style-type: none"> Linear equations Quadratic equations (by factorisation) 				25%
04/3 – 08/3	Equations and Inequalities	<ul style="list-style-type: none"> Quadratic equations (by factorisation) Literal equations (changing the subject of the formula) 				28.1%
11/3 – 15/3	Equations and Inequalities	<ul style="list-style-type: none"> Simultaneous linear equations in two unknowns Solve linear inequalities (show solutions graphically). Interval notation must be known. Word problems involving linear, quadratic or simultaneous linear equations.	F		TEST	31.3%
18/3 – 20/3 (3 days)	Trigonometry	<ul style="list-style-type: none"> Definitions of the trigonometric ratios $\sin \theta$, $\cos \theta$ and $\tan \theta$ in a right-angled triangles. Extend the definitions of $\sin \theta$, $\cos \theta$ and $\tan \theta$ to $0^\circ \leq \theta \leq 360^\circ$.				34.4%
TERM 2			2 TASKS FOR TERM 2			
09/4 – 12/4 (4 days)	Trigonometry	Extend the definitions of $\sin \theta$, $\cos \theta$ and $\tan \theta$ to $0^\circ \leq \theta \leq 360^\circ$. Derive and use values of the trigonometric ratios (without using a calculator for the special angles $\theta \in \{0^\circ; 30^\circ; 45^\circ; 60^\circ; 90^\circ\}$)				37.5%
15/4 - 19/4	Trigonometry	Derive and use values of the trigonometric ratios (without using a calculator for the special angles $\theta \in \{0^\circ; 30^\circ; 45^\circ; 60^\circ; 90^\circ\}$) Solve simple trig equations for $\theta \in \{0^\circ; 90^\circ\}$ Define the reciprocals of trigonometric ratios			ASSIGNMENT OR	40.6%
22/4 – 26/4	Functions	<ul style="list-style-type: none"> Relationships and conversions between variables: numerical, graphical, verbal and symbolical Difference between a relation and a function Investigate basic graphs to discover shape, domain, range, intercepts with axes, turning points and axes of symmetry. Investigate the effect of a and q on each graph Straight line: $y = a(x) + q$ Parabola: $y = a(x)^2 + q$ 				43.8%
29/4 - 03/5 (4 days)	Functions	Hyperbola: Exponential graph: $y = a.b^x + q$; $b > 0$				46.9%
06/5 – 10/5	Functions	<ul style="list-style-type: none"> Finding equations of functions Interpretation of functions 				50%

13/5 - 17/5	Trig Functions	<ul style="list-style-type: none"> • Trig graphs [$a \sin x + q$, $a \cos x + q$, $a \tan x + q$] • Basic graphs and the effect of a and q on the graphs 				53.1%
20/5 – 24/5	Euclidean Geometry	Revise basic results established in earlier grades. Lines, angles, congruency, similarity <ul style="list-style-type: none"> • Investigate line segments joining the midpoints of two sides of a triangle. • Properties of special quadrilaterals. 				56.2%
27/5 - 31/5	Euclidean Geometry	<ul style="list-style-type: none"> • Investigate and make conjectures about the properties of the sides, angles, diagonals and areas of special quadrilaterals. 				59.4%
03/6 – 07/6	JUNE EXAMS			June Exams		
10/6 – 14/6	JUNE EXAMS		F			
17/6 – 21/6	JUNE EXAMS					